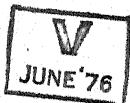


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VIBRATION AND BUCKLING OF GENERALLY ORTHOTROPIC PLATES

A thesis submitted
in partial fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY



By
D. AMRIT SWARUP

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CERTIFIED that this work has been carried out
under my supervision and that it has not been submitted
else-where for a Degree.

(Signature)
(Dr. V. SUNDARAJAN)

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NOTATION

x, y, z	Rectangular coordinates
a	Length of plate along x -axis
b	Width of plate along y -axis
h	Thickness of plate along z -axis
u, v, w	Components of displacements along x, y and z directions.
s_1, s_2, s_3	Strains along x, y , and z directions.
T_q ($q = 1, 2 \dots 6$)	Normal and shear stresses defined in equation A-3.
b_{qr} ($q, r = 1, 2 \dots 6$)	Elastic stiffness of generally orthotropic plate defined by equation A-2.
M_1, M_2, M_{12}, M_{21}	Bending and Twisting moments per unit length.
N_1, N_2	Inplane loads parallel to x and y directions per unit length.
Q_1, Q_2	Plate shears
D_{ij} ($i, j = 1, 2 \dots 6$)	Flexural and twisting rigidities defined by equation 2.7
δ	Mass per unit area of the plate $\times \rho h$
γ	Poissons ratio
p	Inertia force acting on unit area of plate
ω	$\sqrt{\omega}$
ν	Circular frequency
t	Time
C_{mn}	Coefficients in series expansion of plate deflection
ϵ	Angle of orthotropy
U	Total potential energy of plate

R_1, R_2, R_{12}

Non-dimensional inplane load coefficients

$$D_{120}^i = D_{12}^i + 2 D_{66}^i$$

Z

Non-dimensional frequency parameter
 $\approx (\omega^2 + a^4 / \pi^4 D_{11}^i)^{1/2}$

R

Side ratio (a/b)

I

Unitary matrix

C

Column matrix of coefficients Can

g

Beam characteristic function

B_0, I_0

Constants in beam characteristic functions

l

Length of beam

C_{110}, C_{220}

}

Definite integrals equation 2.

D_{110}, D_{220}

}

E_{110}, E_{220}

}

F_{110}, F_{220}

λ

Non-dimensional frequency parameter

$$\approx (\pi^2 \omega^2 / \pi^4 D_{110}^i)^{1/2}$$

X_0, X_1

Non-dimensional normal and shear buckling parameters

ABSTRACT

VIBRATION AND BUCKLING OF GENERALLY ORTHOTROPIC PLATES

by

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SEPTEMBER, 1970.

Vibration and buckling characteristics of thin, rectangular plates, with arbitrary orientation of orthotropy are studied. (Deflections are assumed to be small and the effects of shear deformation and rotatory inertia are neglected.) Approximate solution of the governing differential equation is obtained by the principle of minimum potential energy using a 16 mode Ritz procedure. Beam characteristic functions have been used as "admissible functions" to represent the plate deflections. The required integrals of these functions are evaluated and presented. (The variational equations so obtained are general in nature and are used to find the non-dimensional frequency and critical buckling load parameters of plates with different combinations of simply supported, clamped and free edges, by solving the corresponding eigenvalue problem.) Numerical results of natural frequency and buckling loads are presented for Maple plywood plates having various arbitrary boundary conditions at the edges, different side ratios and angles of orthotropy. The

results for the specially orthotropic case ($\theta = 0^\circ$ or 90°) are compared with previously published results and are found to be in close agreement. The stability and vibration characteristics of a simply supported plate with uniform normal inplane load on a pair of edges is studied. The buckling of simply supported plate subjected to shear loads at the edges is investigated and numerical results are presented for a Mahagony plywood plate.

Graphs are included to show the effect the angle of orthotropicity of the plate on the frequency and critical buckling loads.

INTRODUCTION

1.1 GENERAL

Orthotropic materials play an important role in modern technology. The conflicting demands of increase in strength and stiffness on the one hand and reduction in weight on the other, have led to the use of laminated, stiffened or reinforced construction. Such structural elements are extensively being used in aircraft, missile and ship construction. In the past, materials, regardless of their composition and construction, were generally assumed to be homogeneous and isotropic because of the resulting simplification in the analysis. The present day sophisticated technology, however, requires that the static and dynamic behaviour of orthotropic structures be analysed fully. While certain materials like wood, are orthotropic by nature, a great variety of built up plate-like structures exhibit artificial orthotropicity. Such materials have different elastic properties in different directions. They have, however, three mutually perpendicular planes of elastic symmetry. In an orthotropic plate one of the planes of symmetry is parallel to the plane of the plate. A rectangular plate is "specially orthotropic" if its sides are parallel to the remaining two planes of elastic symmetry, otherwise it is termed as "generally orthotropic". While the isotropic and specially orthotropic plates have received considerable attention of several authors, comparatively less work has been done in the case of

generally orthotropic plate.

1.2 SURVEY OF LITERATURE

a. Vibration Problem:

The transverse vibration characteristics of thin rectangular isotropic plates were analysed and reported in the past. Timoshenko (1)* developed the expression for the potential energy of bending and derived the governing differential equation based on the small deflection thin plate theory. Exact solutions of the differential equation exist when -

- i) all edges are simply supported and
- ii) a pair of opposite edges simply supported with arbitrary boundary conditions at the other edges (method of Levy).

Warburton (2) determined approximate frequencies of rectangular isotropic plates, subjected to different combinations of free, supported or clamped edges. He applied the Rayleigh method*, representing the deflections by suitable characteristic functions, which satisfy the boundary conditions. This method, however, yields the frequencies which are higher than the exact values. Young (3) used the Raleigh Ritz's method to find the approximate natural frequencies of isotropic rectangular plates by using the beam characteristic functions as "admissible" functions to represent the deflection. Numerical results were given

*Numbers in the parentheses designate references at the end of the Thesis.

**In this method the frequency is obtained by equating the maximum kinetic energy of the system to its maximum potential energy.

for square plates with

- i) all edges clamped,
- ii) one edge fixed and remaining free (cantilever),
and iii) two adjacent edges clamped and the remaining free.

Barton (22) extended the treatment of Young to rectangular and skew cantilever plates with various side ratios. He also verified the results experimentally. The experimental values of frequencies were found to be less than those determined from theoretical analysis.

Lekhnitski (24) derived the basic equations of anisotropic elasticity and has considered at length many problems of stress distribution and deformation. Hearmon (5) considered the Hooke's law in its most general form and gave the expressions for the stiffnesses and compliances of orthotropic plates in any arbitrary direction by coordinate transformation. Hearmon and Adams (6) compared the deflection pattern of thin rectangular plates of metal and plywood (cut at various angles to the grain direction) as found from experiments with that from the theoretical analysis, when they are subjected to uniform bending and/or twisting moments at the edges. The results support the theory of bending and twisting of orthotropic plates. Hopmann and Baltimore (7) proposed that an isotropic plate stiffened by ribs can be treated as a homogeneous orthotropic plate by finding the equivalent elastic constants. Knowing these elastic moduli it is possible to predict the behaviour of the plate subjected to any boundary conditions. The experimental results were in close agreement with the

theoretically predicted values.

Huffington and Hoppmann (4) used the Levy's method to find the natural frequency of rectangular, thin, specially orthotropic plates with a pair of opposite edges simply supported and with arbitrary boundary conditions on the remaining edges. Frequency equations and modal eigen functions were derived. The arbitrary boundary conditions included various combinations of free, supported, clamped or elastically restrained edges. Hearmon (8) used the Raleigh's method to find the approximate frequencies of vibration of specially orthotropic plates under different combinations of clamped or supported edges. Closed form expressions were derived for the frequencies using beam characteristic functions to represent the deflection. Numerical results of the fundamental frequency of square plates were given for six combinations of supported and clamped edges. Somayajulu and Srinivasan (9) extended the method of Huffington and Hoppmann to find the first six frequencies of vibration of specially orthotropic plates with different side ratios. They, further, used the Raleigh Ritz method to determine the first five frequencies of vibration of specially orthotropic cantilever plates. Numerical results of these frequencies were given for five materials with different orthotropic properties and side ratios.

Calligeros and Dugundji (10) investigated the supersonic flutter of generally orthotropic panels using the principle of minimum potential energy and the Raleigh-Ritz method. They have plotted the frequencies of natural vibration

for the first sixteen modes of such panels with three different side ratios and two sets of orthotropic properties.

Weeks and Shidler (11) considered the vibration characteristics of thin, rectangular, specially orthotropic plates, with inplane loads, subjected to different combinations of supported, clamped and elastically restrained edge conditions. The Galerkin's method was used to derive the frequency equations. Dickinson (12) used a sine series solution to analyse the free vibration of specially orthotropic, thin rectangular plates. The method of solution is applicable to plates with any edge conditions expressible using sine series. Application to plates with the following boundary conditions was given :

- i) A pair of opposite edges simply supported and each of the remaining edges being simply supported, free, clamped or partially restrained,
- ii) all edges clamped and
- iii) two opposite edges clamped and the remaining edges free.

Numerical results were given for square plates with (i) all edges clamped and (ii) two opposite edges clamped and the remaining being free. The results were compared with those already available in the literature. The accuracy of the numerical results depends on the convergence of the roots of the determinantal equation.

b. Stability Problem:

The buckling of isotropic plates was discussed at length by Timoshenko and Gere (13). The governing differential

and

equation/expressions for potential energy were derived. Exact solutions for the critical buckling loads were given for plates with all edges simply supported. The governing differential equation was also solved for plates with the pair of loaded edges simply supported and with arbitrary boundary conditions at the remaining edges, using Levy's method. Maulbetsch (14) used the energy method* to find the approximate critical buckling loads of rectangular isotropic plates with clamped edges. Levy (15) obtained an exact solution of the governing differential equation in terms of infinite series to get the critical buckling loads of isotropic plates with clamped edges. The accuracy of the results depends on the convergence of the series and he estimated that the error is of the order of 0.1%. Green and Hearman (16) derived the differential equation of bending of thin, rectangular, generally orthotropic plates with uniform inplane loads. Using Fourier series method, the cases of a plate with (i) all edges simply supported and (ii) all edges clamped were solved. Results were also given for a plate with a pair of opposite edges simply supported, the remaining being clamped. Numerical values of critical axial and shear buckling loads were evaluated for square and infinitely long generally orthotropic plates with support conditions (i) and (ii). They remark that the results obtained are reasonably accurate for square plates but as the side ratio (a/b) increases the accuracy diminishes.

estimating the potential energy.

Das (17) used the Levy's method to find the critical buckling loads of thin rectangular specially orthotropic plates with the pair of loaded edges simply supported and the remaining edges having arbitrary boundary conditions. Numerical results were given for different types of plywood plates under the following boundary conditions :

- i) all edges simply supported
- ii) three edges simply supported and the fourth clamped and
- iii) three edges simply supported and the fourth free.

Lure (18) has observed that the vibration as well as the buckling analysis of thin rectangular plates leads to the same Eigenvalue problem under certain boundary conditions. He has suggested a method of finding the frequency of natural vibrations from the critical buckling load parameters. Somayajulu and Srinivasan (9) used this analogy to find the critical buckling loads from the frequency data. Weeks and Shidell (11) calculated the buckling characteristics of specially orthotropic plates by noting the fact that the critical buckling load is the lowest inplane load which makes the frequency of transverse vibration of the plate (subjected to inplane load) vanish.

1.3 STATEMENT OF THE PROBLEM

In the present investigation the vibration and buckling characteristics of thin, rectangular generally orthotropic plates are studied. The material is assumed to be linearly elastic and the analysis is based on the small deflection theory

of thin plates. Effects of shear deformation and rotatory inertia are neglected. A sixteen node Raleigh Ritz procedure along with the principle of minimum potential energy is used to get an approximate solution of the governing differential equation. Beam characteristic functions, which represent the normal modes of vibration of slender beams, are used as "admissible" functions.

Integrals of these functions are evaluated using a numerical integration scheme. A general formulation of the problem was obtained which could be used to find the frequencies of natural vibration as well as the critical buckling loads (shear and normal) of thin rectangular plates with arbitrary orientation of orthotropicity and any boundary conditions at the edges.

The above procedure is applied to find the non-dimensional frequency and critical buckling load parameters of rectangular generally orthotropic plates having the following edge conditions :

- 1) all edges simply supported,
- ii) all edges clamped,
- iii) one edge clamped and the rest free (cantilever plate),
- iv) three edges simply supported and the remaining free and
- v) a pair of opposite edges simply supported and the rest clamped.

FORMULATION AND SOLUTION

2.1 GENERAL EQUATIONS FOR ORTHOTROPIC PLATES

The governing differential equation and the boundary conditions of a generally orthotropic, thin, rectangular plate are derived by applying the principle of minimum potential energy. The plate (figure No.1) is assumed to have three axes of elastic symmetry, one at right angles to the plane of the plate and the other two lying in its plane, making an angle θ with the sides. The small deflection theory of thin plates is utilized in the analysis which is based on the following assumptions.

1. Thickness of the plate is small when compared to its lateral dimensions.

2. Deflection of the middle surface is small when compared to the thickness of the plate.

3. Rectilinear sections which in the undeformed plate were normal to the middle surface remain rectilinear and normal to the bent middle surface and

4. Normal stress T_3 on planes parallel to the middle surface is small in comparison with the stresses T_1 , T_2 and T_6 acting in its plane.

Under these assumptions the displacements are linearly related to the distance z from the middle surface of the plate

and are given by

$$u = -z \frac{\partial v}{\partial x} ; \quad v = -z \frac{\partial u}{\partial y}$$

the strains in the plane of the plate are given by

$$\begin{aligned} s_1 &= \frac{\partial u}{\partial x} = -z \frac{\partial^2 v}{\partial x^2} \\ s_2 &= \frac{\partial v}{\partial y} = -z \frac{\partial^2 u}{\partial y^2} \\ s_6 &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -2z \frac{\partial^2 v}{\partial x \partial y} \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \quad \dots 2.1$$

The stress displacement relations are (from eqn. A.5)

$$\begin{aligned} T_1 &= -z \left[b_{11} \frac{\partial^2 v}{\partial x^2} + b_{12} \frac{\partial^2 v}{\partial y^2} + 2b_{16} \frac{\partial^2 v}{\partial x \partial y} \right] \\ T_2 &= -z \left[b_{12} \frac{\partial^2 v}{\partial x^2} + b_{22} \frac{\partial^2 v}{\partial y^2} + 2b_{26} \frac{\partial^2 v}{\partial x \partial y} \right] \\ T_3 &= -z \left[b_{16} \frac{\partial^2 v}{\partial x^2} + b_{26} \frac{\partial^2 v}{\partial y^2} + 2b_{66} \frac{\partial^2 v}{\partial x \partial y} \right] \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \quad \dots 2.2$$

The stress components T_4 and T_5 are given by (ref. 5)

$$\begin{aligned} T_4 &= \frac{1}{2} \left(z^2 - \frac{h^2}{4} \right) \left[b_{16} \frac{\partial^2 v}{\partial x^2} + (b_{12} + 2b_{66}) \frac{\partial^2 v}{\partial x \partial y} + 2b_{26} \frac{\partial^2 v}{\partial y^2} \right. \\ &\quad \left. + b_{22} \frac{\partial^2 v}{\partial y^2} \right] \\ T_5 &= \frac{1}{2} \left(z^2 - \frac{h^2}{4} \right) \left[b_{11} \frac{\partial^2 v}{\partial x^2} + 2b_{16} \frac{\partial^2 v}{\partial x^2 \partial y} + (b_{12} + 2b_{66}) \frac{\partial^2 v}{\partial x \partial y^2} \right. \\ &\quad \left. + b_{26} \frac{\partial^2 v}{\partial y^2} \right] \quad \dots 2.3 \end{aligned}$$

The bending moments, twisting moments and inplane loads per unit length are given by

$$M_1 = \int_{-h/2}^{h/2} T_1 \, dz ; \quad M_2 = \int_{-h/2}^{h/2} T_2 \, dz ; \quad M_{12} = \int_{-h/2}^{h/2} T_6 \, dz$$

$$M_1 = \int_{-h/2}^{h/2} T_5 \, dz \text{ and } M_2 = \int_{-h/2}^{h/2} T_4 \, dz \quad \dots 2.4$$

where h is the thickness of the plate.

Substituting equations 2.2 and integrating we get

$$\begin{aligned} M_1 &= - (D_{11} \frac{\partial^2 v}{\partial x^2} + D_{12} \frac{\partial^2 v}{\partial y^2} + 2 D_{16} \frac{\partial^2 v}{\partial x \partial y}) \\ M_2 &= - (D_{12} \frac{\partial^2 v}{\partial x^2} + D_{22} \frac{\partial^2 v}{\partial y^2} + 2 D_{26} \frac{\partial^2 v}{\partial x \partial y}) \quad \dots 2.5 \\ M_{12} &= (D_{16} \frac{\partial^2 v}{\partial x^2} + D_{26} \frac{\partial^2 v}{\partial y^2} + 2 D_{66} \frac{\partial^2 v}{\partial x \partial y}) \end{aligned}$$

and the plate shears are given by (from equation 2.3)

$$\begin{aligned} Q_1 &= - \left[D_{11} \frac{\partial^2 v}{\partial x^2} + 2 D_{16} \frac{\partial^2 v}{\partial x \partial y} + (D_{12} + 2D_{66}) \frac{\partial^3 v}{\partial x^2 \partial y} + D_{26} \frac{\partial^3 v}{\partial y^3} \right] \\ Q_2 &= - \left[D_{16} \frac{\partial^2 v}{\partial x^2} + (D_{12} + 2D_{66}) \frac{\partial^2 v}{\partial x^2 \partial y} + 3D_{26} \frac{\partial^3 v}{\partial x \partial y^2} + D_{22} \frac{\partial^3 v}{\partial y^3} \right] \quad \dots 2.6 \end{aligned}$$

$$\text{where } D_{ij} = b_{ij} \frac{h^3}{12} \quad \dots 2.7$$

The above equations for the specially orthotropic case are obtained by putting $b_{16} = b_{26} = 0$ and for the isotropic case by putting $b_{11} = b_{22} = E/(3-\gamma^2)$ and $b_{66} = 0$.

Substituting equations 2.9 in 2.7 the rigidities of a generally orthotropic plate (D_{ij}) with an angle of orthotropy θ , can be expressed in terms of the rigidities of a specially orthotropic plate by a set of equations obtained by replacing b_{ij}

by D_{1j} and b_{1j} by D_{1j}' in equations 2.7 where D_{1j}' are the rigidities of a specially orthotropic plate.

The total potential energy (U) of a thin, rectangular, plate of uniform thickness h undergoing transverse vibrations is given by (ref. 19).

$$U = \frac{1}{2} \int_{-h/2}^{h/2} \int_0^a \int_0^b (T_1 S_1 + T_2 S_2 + T_3 S_3) dx dy dz$$

$$- \frac{1}{2} \int_0^a \int_0^b \left[pw - N_1 \left(\frac{\partial w}{\partial x} \right)^2 - N_2 \left(\frac{\partial w}{\partial y} \right)^2 - 2N_{12} \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right] dx dy \quad \dots 2.8$$

Substituting the stress-strain (1.5) and the strain-displacement (2.1) relations we get

$$U = \frac{1}{2} \int_0^a \int_0^b \left[D_{11} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + 2 D_{12} \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + D_{22} \left(\frac{\partial^2 w}{\partial y^2} \right)^2 \right. \\ \left. + 4 D_{33} \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 + 4 \frac{\partial^2 w}{\partial x \partial y} (D_{13} \frac{\partial^2 w}{\partial x^2} + D_{23} \frac{\partial^2 w}{\partial y^2}) + pw \right. \\ \left. + N_1 \left(\frac{\partial w}{\partial x} \right)^2 + N_2 \left(\frac{\partial w}{\partial y} \right)^2 + 2N_{12} \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right] dx dy \quad \dots 2.9$$

$$\text{where } D_{1j} = \int_{-h/2}^{h/2} D_{1j} s^2 dz = D_{1j} h^3/12$$

The differential equation and the boundary conditions of the plate can be obtained by applying the "Principle of minimum potential Energy". Taking the variation of equation 2.8 and performing the integration by parts, we get

$$U = \iint_A \left[D_{11} \frac{\partial^4 v}{\partial x^4} + 2 D_{12} \frac{\partial^4 v}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 v}{\partial y^4} + 4 D_{23} \frac{\partial^4 v}{\partial x^2 \partial y^2} \right] \delta v \, dx \, dy$$

$$+ 4 D_{23} \frac{\partial^4 v}{\partial x \partial y^3} - p - N_1 \frac{\partial^2 v}{\partial x^2} - N_2 \frac{\partial^2 v}{\partial y^2}$$

$$- 2 N_{12} \frac{\partial^2 v}{\partial x \partial y} \right] \delta v \, dx \, dy$$

$$- \int_0^b N_1 \delta \left(\frac{\partial v}{\partial x} \right) \Big|_0^a \, dy = \int_0^a N_2 \delta \left(\frac{\partial v}{\partial y} \right) \Big|_0^b \, dx$$

$$+ \int_0^b \left[Q_1 + \frac{\partial (N_{12})}{\partial y} + N_2 \frac{\partial v}{\partial x} + N_{12} \frac{\partial v}{\partial y} \right] \delta v \, dy \Big|_0^a$$

$$+ \int_0^a \left[Q_2 + \frac{\partial (N_{12})}{\partial x} + N_2 \frac{\partial v}{\partial y} + N_{12} \frac{\partial v}{\partial x} \right] \delta v \, dx \Big|_0^b$$

$$- 2 \left[N_{12} \delta v \right] \Big|_0^a \Big|_0^b = 0$$

... 2.10

where N_1 , N_2 , N_{12} , Q_1 and Q_2 have the same meaning as in equations 2.6 and 2.6. Since $\delta v(x, y, t)$ is arbitrary, equation 2.10 will be satisfied if the following conditions hold.

$$1) D_{11} \frac{\partial^4 v}{\partial x^4} + 2 D_{12} \frac{\partial^4 v}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 v}{\partial y^4} + 4 D_{23} \frac{\partial^4 v}{\partial x^2 \partial y^2}$$

$$+ 4 D_{23} \frac{\partial^4 v}{\partial x \partial y^3} - p - N_1 \frac{\partial^2 v}{\partial x^2} - N_2 \frac{\partial^2 v}{\partial y^2} - 2 N_{12} \frac{\partial^2 v}{\partial x \partial y} = 0$$

... 2.11

This is the governing differential equation of a vibrating plate subjected to inplane loads N_1 , N_2 and N_{12} as shown in

figure 1.

11)

a) Along edges $x = 0, a$

$$(1) \text{ Either } M_1 = 0 \text{ or } \delta \left(\frac{\partial w}{\partial x} \right) = 0$$

$$(2) \text{ Either } Q_1 + \frac{\partial (M_{12})}{\partial y} + M_1 \frac{\partial w}{\partial x} + M_{12} \frac{\partial w}{\partial y} = 0 \text{ or } \delta w = 0$$

b) Along edges $y = 0, b$

$$(1) \text{ Either } M_2 = 0 \text{ or } \delta \left(\frac{\partial w}{\partial y} \right) = 0$$

$$(2) \text{ Either } Q_2 + \frac{\partial (M_{12})}{\partial x} + M_2 \frac{\partial w}{\partial y} + M_{12} \frac{\partial w}{\partial x} = 0 \text{ or } w = 0$$

c) At corners

$$\text{Either } M_{12} = 0 \text{ or } \delta w = 0$$

... 2.12

This condition can be physically interpreted to give the boundary conditions of the plate at the edges.

For a specially orthotropic plate the terms containing D_{36} and D_{26} vanish and the governing differential equation (2.10) is simplified accordingly.

For the isotropic plate with no inplane loads the equation reduces to the well known form

$$\nabla^4 w = \frac{p}{D}$$

where $D = E h^3 / 12(1-\nu^2)$

2.2 FREQUENCY OF TRANSVERSE VIBRATION

For a plate undergoing transverse vibrations without any external loads the governing differential equation is obtained by replacing

$$p \text{ by } -\delta \frac{\partial^2 v}{\partial t^2} \text{ and substituting } N_{12}N_2 = N_{12} = 0$$

in equation 2.11. It then becomes

$$\begin{aligned} D_{11} \frac{\partial^4 v}{\partial x^4} + 2D_{12} \frac{\partial^4 v}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 v}{\partial y^4} + 4D_{16} \frac{\partial^4 v}{\partial x^2 \partial y^2} \\ + 4D_{26} \frac{\partial^4 v}{\partial y^2 \partial z^2} + \delta \frac{\partial^2 v}{\partial t^2} = 0 \end{aligned} \quad \dots 2.13$$

This equation is difficult to be solved in most cases and exact solutions are known only for some simple cases. For the case of an isotropic or specially orthotropic plate with all edges simply supported, the equation has been solved (ref. 1 and 19). Exact solution also exists when a pair of opposite edges are simply supported for these plates (Lévy's method). In this method the deflection is assumed to be

$$v = f(y) \text{ in } \frac{\pi \pi}{a} e^{i\omega t} \quad \dots 2.14$$

equation 2.14 when substituted in the partial differential equation will convert it into an ordinary differential equation. The frequency equation can then be derived by solving this equation. Huffington and Hoffmann (4) have used this method for specially orthotropic plates.

For other boundary conditions of isotropic and specially orthotropic plates and any boundary conditions of generally

orthotropic plates it is difficult to solve the differential equation exactly. Under these circumstances we have to take recourse to approximate methods. Raleigh's method was used by Warburton (2) for isotropic plates and by Hoppmann and Baltimore (7) for specially orthotropic plates. This method, however, cannot be applied to the generally orthotropic plates because of the occurrence of terms containing D_{15} and D_{26} . For the same reason Galerkin's method also cannot be applied for this case. In the present study the Raleigh Ritz method is used to obtain approximate solution to the governing differential equation. This method was used by several previous authors. Young (3) applied it to isotropic plates, Somayajulu and Srinivasan (9) for specially orthotropic plates and Calligerous and Dugundji (10) to the analysis of orthotropic panel flutter.

2.3 THE RALEIGH RITZ METHOD.

The Raleigh Ritz method, in effect, substitutes a variational problem for the usual characteristic value problem. In a variational problem it is not necessary to impose boundary conditions in advance, in order to single out a specific solution. The vanishing of the first variation of the functional not only implies the Euler's equations, but also the natural boundary conditions. Courant (22) remarks that for rigid boundary conditions the approximation of the Raleigh Ritz method is good and a few admissible coordinate functions would in most cases, suffice to yield the desired convergence. But the boundary conditions impose a restriction on the choice of functions to represent the deflection. For free boundaries, however, the choice of functions is unlimited, but the convergence is rather slow and it

becomes necessary to take more terms in the series to get the desired accuracy.

In applying the Raleigh Ritz method, the deflection of the plate is assumed as a linear series of admissible coordinate functions. The coefficient of each term of the series is adjusted so as to minimize the potential energy (U) of the plate. The deflection W of the plate can be assumed as

$$W(x, y, t) = \sum_{m=1}^p \sum_{n=1}^q C_{mn} X_m(x) Y_n(y) e^{i\omega t} \quad \dots 2.15$$

where the functions X_m and Y_n are "admissible" functions i.e. they satisfy the artificial (or rigid) boundary conditions and need not satisfy the natural boundary conditions. In the case of plates prescribed values of slope and deflection constitute the artificial boundary conditions and the values of moments and shear force constitute the natural boundary conditions. Better convergence, however, can be obtained if the natural boundary conditions are also satisfied by the assumed functions.

By substituting equation 2.15 in equation 2.9 the total Potential Energy U can be expressed as a function of the coefficients C_{mn}

$$U = f(C_{11}, C_{12}, \dots, C_{pq})$$

for minimum potential energy we have $\delta U = 0$

$$\text{or } \sum_{m=1}^p \sum_{n=1}^q \frac{\partial U}{\partial C_{mn}} \delta C_{mn} = 0$$

$$\text{i.e., } \frac{\partial^2 v}{\partial x^2} = 0, (m = 1, 2, \dots, p; n = 1, 2, \dots, q) \quad \dots \quad 2.16$$

Equations 2.16 represent a system of homogeneous, linear equations in the unknown quantities C_{mn} . There are $p \times q$ equations in $p \times q$ unknowns, which can be determined by equating the determinant of the coefficient matrix to zero. The Eigenvalues of the system can be determined by any numerical technique, using a high speed electronic digital computer.

2.4 PLATE SIMPLY SUPPORTED ON ALL EDGES.

For a plate with all edges simply supported the deflection can be assumed as

$$v(x, y, t) = \sum_{m=1}^p \sum_{n=1}^q C_{mn} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} e^{i \omega t} \quad \dots \quad 2.17$$

This satisfies the geometric boundary conditions

$$v(x, 0, t) = v(x, b, t) = v(0, y, t) = v(a, y, t) = 0$$

But, the natural boundary conditions

$$\left[D_{11} \frac{\partial^2 v}{\partial x^2} + 2 D_{12} \frac{\partial^2 v}{\partial x \partial y} + D_{22} \frac{\partial^2 v}{\partial y^2} \right] \begin{array}{l} x = a \\ x = 0 \end{array} = 0 \text{ and}$$

$$\left[D_{22} \frac{\partial^2 v}{\partial y^2} + 2 D_{12} \frac{\partial^2 v}{\partial x \partial y} + D_{11} \frac{\partial^2 v}{\partial x^2} \right] \begin{array}{l} y = b \\ y = 0 \end{array} = 0$$

are not satisfied.

Substituting 2.17 in 2.9 and noting that for the

vibration problem $p = -\frac{\partial^2 w}{\partial t^2}$ we get, after simplification,

$$U = \frac{1}{2} \left[\left(\frac{D_{11} m^4 \pi^4}{a^4} + \frac{2 D_{12} m^2 n^2 \pi^4}{a^2 b^2} + \frac{D_{22} n^4 \pi^4}{b^4} \right) \frac{\partial^2 w}{\partial t^2} \right]$$

$$\int_0^a \int_0^b \left(\sum \sum c_{mn} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \right)^2 dx dy$$

$$+ \frac{4 m^2 n^2 \pi^4 D_{66}}{a^2 b^2} \int_0^a \int_0^b \left(\sum \sum c_{mn} \cos \frac{m \pi x}{a} \cos \frac{n \pi y}{b} \right)^2 dx dy$$

$$+ \frac{N_1 m^2 \pi^2}{a^2} \int_0^a \int_0^b \left(\sum \sum c_{mn} \cos \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \right)^2 dx dy$$

$$+ \frac{N_2 n^2 \pi^2}{b^2} \int_0^a \int_0^b \left(\sum \sum c_{mn} \sin \frac{m \pi x}{a} \cos \frac{n \pi y}{b} \right)^2 dx dy$$

$$+ \frac{2 N_{12} m n \pi^2}{ab} \int_0^a \int_0^b \left(\sum \sum c_{mn} \cos \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \right)$$

$$\left(\sum \sum c_{mn} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \right) dx dy - \left(\frac{4 D_{16} m^3 n \pi^4}{a^3 b} \right)$$

$$+ \frac{4 D_{26} m n^3 \pi^4}{a b^3} \int_0^a \int_0^b$$

$$\left[\sum \sum c_{mn} \cos \frac{m \pi x}{a} \cos \frac{n \pi y}{b} \right] \left(\sum \sum c_{mn} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \right) e^{2i\omega t} \dots$$

By substituting equation 2.18 in equation 2.16, carrying out the differentiation, simplifying and using the following integrals

$$\int_{-a}^a \sin \frac{m\pi x}{a} \sin \frac{n\pi x}{a} dx = \int_{-a}^a \cos \frac{m\pi x}{a} \cos \frac{n\pi x}{a} dx$$

$$= 0 \text{ if } m \neq n, = \frac{a}{2} \text{ if } m = n$$

and $\int_{-a}^a \sin \frac{m\pi x}{a} \cos \frac{n\pi x}{a} dx = 0 \text{ if } m + n \text{ is even}$

$$= \frac{2a n}{\pi(m^2 - n^2)} \text{ if } m + n \text{ is odd}$$

we get

$$\left[u^4 \frac{D_{11}}{D'_{11}} + 2 R^2 \frac{D_{126}}{D'_{11}} (mn)^2 + \frac{R^4 D_{22} u^4}{D'_{11}} + R_1 u^2 + R_2 u^2 - Z \right] C_{mn}$$

$$= 2 \sum_{m=1}^p \sum_{n=1}^q C_{mn} \left[\frac{R D_{126} (m^2 + n^2)}{D'_{11}} + \frac{R^2 D_{22} (m^2 + n^2)}{D'_{11}} + R R_{12} \right]$$

$$R_{12} = 0$$

$$(m = 1, 2, \dots, p; n = 1, 2, \dots, q) \quad \dots \quad 2.19$$

where

$$D_{126} = D_{22} + 2 R_{66}, \quad Z = u^2 + u^4 / \pi^2 D'_{11}$$

$$R_1 = R_1 u^2 / \pi^2 D'_{11}, \quad R_2 = R_2 u^2 / \pi^2 D'_{11}$$

$$R_{12} = R_{12} u^2 / \pi^2 D'_{11}, \quad R = a/b \text{ (side ratio)}$$

and

$$K_{mnr} = \frac{16 \text{ mnr}}{\pi^2 (r^2 - m^2) (s^2 - n^2)} \quad \text{when } m \pm r \text{ is odd}$$

and $n \pm s$ is odd

$$\pm 0, \text{ when } m \pm r \text{ is even}$$

For the free vibration problem ($R_1 = R_2 = R_{12} = 0$), it is convenient to write the set of equations in the matrix form

$$[K] \{c\} - \omega^2 [I] \{c\} = 0 \quad \dots 2.20$$

$[K]$ will be a diagonal matrix when $\theta = 0$ or 90° and for other angles it will be real, symmetric and positive definite. The Eigenvalues of $[K]$ which will be real and positive represent the natural frequencies of transverse wave vibration of the plate. The Jacobi's method is used for solving the eigenvalue problem represented by 2.20. This is a method of diagonalization by successive rotations and iterates to all eigenvectors and eigenvalues simultaneously. This procedure consists of multiplications by matrices, which are similar in form to coordinate transformation matrices that represent angular rotations. The successive multiplications result in the gradual increase of the diagonal terms at the expense of the off-diagonal elements. When finally the off-diagonal elements become zero, the diagonal terms of the resulting matrix are the eigenvalues and the continuous product of rotation is the modal matrix. The method is readily applicable to real and symmetric matrices using the

high speed electronic digital computer.

2.5 PLATE WITH ARBITRARY BOUNDARY CONDITIONS.

For a plate with arbitrary boundary conditions at the edges we can assume

$$w(x,y,t) = \sum_{m=1}^p \sum_{n=1}^q c_{mn} X_m(x) Y_n(y) e^{i\omega t} \quad \dots 2.21$$

where X_m and Y_n are both "admissible" functions i.e. they satisfy the rigid boundary conditions. In the present analysis the "beam characteristic functions" are used for these functions. These represent the normal modes of vibration of uniform, long and slender beams. They are used as admissible functions for plate vibration problems by several previous authors Warburton (2) and Young (3) used them for isotropic plates and Hearmon (8) and Somayajulu and Srinivasan (9) for specially orthotropic plates. These functions, because of their orthogonal property, are simple to use and many of the integrals required for the computations for certain boundary conditions are calculated and tabulated by Young (3). The other integrals required for the generally orthotropic plate for all boundary conditions are evaluated and tabulated in the present investigation.

Beam Characteristic Functions:

These functions are obtained from the solution of the differential equation governing the transverse vibration of a uniform beam. The general form of these functions is given by

$$\theta_c = \cosh B_c x + B \cos B_c x - L_c (\sinh B_c x + P \sin B_c x) \quad \dots 2.22$$

These are, thus, an infinite number of functions corresponding to $c = 1, 2, \dots, \infty$ representing the different nodes of vibration. While the equation 2.22 represents the general form of these functions, for any particular boundary conditions of the beam the corresponding values of E, F, B_0 and L_0 are to be substituted. These functions and constants are given by Bishop (21), Hearmon (5) and Young (3). The data compiled from these references is given in table 1 for different boundary conditions.

These functions possess the important property of orthogonality i.e. for any two functions ϕ_m and ϕ_n

$$\int_0^l \phi_m \phi_n dx = \begin{cases} 1 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases} \quad \dots 2.23$$

and

$$\int_0^l \phi_m^{11} \phi_n^{11} dx = \begin{cases} \frac{\pi^4}{l^3} & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases} \quad \dots 2.24$$

where l is the length of the beam

The characteristic functions for a simply supported beam, which are not given in the table, can be taken as

$$\phi_c = \sqrt{\frac{c}{l}} \sin \left(B_0 \frac{x}{l} \right) \quad \dots 2.25$$

These functions also satisfy the orthogonal relations 2.23 and 2.24, if we define $B_0 = c \pi$ ($c = 1, 2, \dots, \infty$).

The lowest mode ($c = 1$) in table corresponds to two nodal lines and when dealing with free-supported and free-free plates modes of lower frequencies corresponding to rigid body translation and rotation are possible. These modes are represented by additional characteristic functions proposed by Warburton (2) -

i) Free-supported plate

$$\phi_1 = \sqrt{3} (1 - x/l) \quad \dots \quad 2.26$$

ii) Free-free Plate

$$\begin{aligned} \phi_1 &= 1.0 \\ \phi_2 &= \sqrt{3} (1 - 2x/l) \quad \dots \quad 2.27 \end{aligned}$$

These functions also satisfy the orthogonal relations represented by equations 2.23 and 2.24 if we define for these lower modes $R_C = 0$ ($c = 1, 2$)

Using these functions for $X_m^{(x)}$ and $Y_n^{(y)}$ in equations 2.21 the total potential energy of the vibrating plate with any combination of free, supported or clamped edges can be expressed in terms of the unknown constants C_{mn} . Substituting 2.21 in equations 2.9 and 2.16, carrying out the differentiation and simplifying using the following notation

$$C_{mn} = a \int_0^a X_m^{(x)} X_n^{(x)} dx$$

$$C_{nn} = b \int_0^b Y_n^{(y)} Y_n^{(y)} dy$$

$$D_{mn} = a \int_0^a X_m^{(x)} X_n^{(x)} dx$$

$$D_{nn} = b \int_0^b Y_n^{(y)} Y_n^{(y)} dy$$

$$E_{\text{MP}} = a \int_0^a X_m'' X_p' dx \quad E_{\text{NS}} = b \int_0^b Y_n'' Y_s' dy$$

$$F_{\text{MP}} = a \int_0^a X_m'' X_p dx \quad F_{\text{NS}} = b \int_0^b Y_n'' Y_s dy$$

... 2.28

$$G_{\text{MPNS}} = F_{\text{MP}} F_{\text{SN}} + F_{\text{PM}} F_{\text{NS}}$$

... 2.29

$$H_{\text{MPNS}} = E_{\text{PM}} C_{\text{NS}} + E_{\text{MP}} C_{\text{SN}}$$

$$P_{\text{MPNS}} = C_{\text{MP}} E_{\text{SN}} + C_{\text{PM}} E_{\text{NS}}$$

get

$$\sum_r \sum_s \left[D_1 B_m^4 S_{\text{MPNS}} + D_2 R^2 G_{\text{MPNS}} + D_3 B_0^4 R^4 S_{\text{MPNS}} \right. \\ + 4 D_4 D_{\text{MP}} D_{\text{NS}} R^2 + 2 D_5 H_{\text{MPNS}} R^2 + 2 D_6 P_{\text{MPNS}} R^2 \\ - \lambda^2 S_{\text{MPNS}} + H_1 a^2 D_{\text{MP}} S_{\text{NS}} / D_{22} + H_2 R^2 a^2 D_{\text{NS}} S_{\text{MP}} / D_{22} \\ \left. + H_{12} a^2 (D_{\text{MP}} S_{\text{NS}} + R^2 D_{\text{NS}} S_{\text{MP}}) / D_{22} \right] \quad C_{rs} = 0$$

... 2.30

where

$$D_1 = D_{11} / D_{22} ; \quad D_2 = D_{22} / D_{22}^2 ; \quad D_3 = D_{12} / D_{22}^2$$

$$D_4 = D_{00} / D_{22}^2 ; \quad D_5 = D_{10} / D_{22}^2 ; \quad D_6 = D_{20} / D_{22}^2$$

$$S_{\text{MPNS}} = 1 \text{ if } n = r \text{ and } n = s \\ = 0 \text{ if } n \neq r \text{ or } n \neq s$$

$$S_{\text{MP}} = 1 \text{ if } n = r \\ = 0 \text{ if } n \neq r$$

$$S_{\text{NS}} = 1 \text{ if } n = s \\ = 0 \text{ if } n \neq s$$

$$\text{and } \lambda^2 = \frac{\rho h^2 a^4}{D_{22}}$$

For the free vibration problem ($N_1 = N_2 = N_{12} = 0$) the equations 2.30 can be written in the matrix form

$$[K] \{c\} = \lambda^2 [I] \{c\} \quad \dots \quad 2.31$$

The matrix $[K]$ is real and symmetric irrespective of the boundary conditions at the edges of the plate. The eigenvalue problem represented by the above equation can be solved to yield the frequencies of transverse vibration of the plate (λ) for any boundary conditions at the edges, angle of orthotropicity (θ) and side ratio (R).

2.6 BUCKLING OF GENERALLY ORTHOTROPIC PLATES

The critical values of forces applied in the middle plane of the plate at which the flat form of equilibrium becomes unstable and the plate begins to buckle can be determined by several methods.

1) The plate is assumed to have initially some curvature or lateral load. The inplane forces required to make deflections tend to grow indefinitely are the critical values (Imperfection method).

2) The plate is assumed to buckle slightly under the action of the middle plane forces and the values of these forces are calculated in order to keep the slightly buckled shape in equilibrium (equilibrium method). The differential equation of the surface is obtained in this case by putting $p = 0$ in equation ^{2.11} and

is given by 2.31

$$\begin{aligned} D_{11} \frac{\partial^4 v}{\partial x^4} + 2D_{12} \frac{\partial^4 v}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 v}{\partial y^4} + 4 D_{12} \frac{\partial^4 v}{\partial x^2 \partial y^2} \\ + 4 D_{22} \frac{\partial^4 v}{\partial x \partial y^3} \\ = N_1 \frac{\partial^2 v}{\partial x^2} + N_2 \frac{\partial^2 v}{\partial y^2} + 2 N_{12} \frac{\partial^2 v}{\partial x \partial y} \quad \dots 2.32 \end{aligned}$$

If only N_1 is acting, the minimum value of N_1 which satisfies the above equation and the boundary conditions of the plate, is the critical buckling load. This method has been used in the literature for the buckling analysis of plates. Green and Beaumont (16) have used this method for 'generally' orthotropic plates employing the Fourier series method to solve the differential equation Das (17) has used this approach to find the critical buckling loads of rectangular, specially orthotropic plates, using the Levy's method to solve equation 2.32.

3) Where a rigorous solution of equation 2.32 is unknown we can calculate the approximate critical buckling loads by equating the strain energy of bending to the work done by the inplane forces (Energy method).

In this method the plate is assumed to undergo small lateral bending consistent with the boundary conditions. If the work done by the inplane forces is smaller than the strain energy of bending for every possible shape of lateral buckling, the flat form of equilibrium is stable. If it is greater than the strain energy of bending the plate is unstable in the flat form

and undergoes buckling. The inplane loads reach critical values when the work done by the inplane forces is equal to the strain energy of bending. Timoshenko and Gere (13) have used this method to find the critical buckling loads for rectangular, isotropic plates with all edges simply supported.

4) The lowest inplane load which makes the frequency of transverse vibration vanish gives the critical buckling load (Kinetic method). This can be obtained from a plot of the frequency of transverse vibration with inplane load versus the magnitude of the inplane load. This method was used by Weeks and Shideler (11) to find the critical buckling loads of specially orthotropic plates.

5) By noting the fact that under certain boundary conditions the buckling and vibration problems are similar boundary value problems, the results of one can be obtained if the results of the other are available. The critical buckling load parameters can be found from the natural frequency parameters by suitable conversion. This analogy can be clearly seen in the case of orthotropic plate with simply supported edges from equation 2.17. For the vibration problem, in the above equations we have $R_1 = R_2 = R_{12} = 0$, whereas for the buckling problem we have $R_2 = R_{12} = \lambda = 0$. The equation can then be put in the matrix form

$$[K] \{d\} = [I] \{c\} \quad \dots 2.39$$

Thus both the problems can be reduced to the same Eigen value problem and the Eigenvalues, λ can be interpreted as the

natural frequencies for the vibration problem and as the critical buckling loads for the buckling problem.

Such a similarity can also be noticed in the case of a plate with the pair of loaded edges simply supported. Again we have for the vibration problem $R_1 = R_2 = R_{12} = 0$ and for the buckling problem $R_2 = R_{12} = 1 = 0$. It can be seen that $D_{11} = (m \pi)^2 \delta_{11}$ and the eigenvalue problem can be put in the matrix notation (equation 2.33). The Eigenvalues can be suitable interpreted as

$$Z = \left(\frac{\pi^2 a^4 / D_{22}}{2} \right)^{1/2} \text{ for the vibration problem}$$

$$= N_1 a^2 m^2 \pi^2 / D_{22}^2 \text{ for the buckling problem.}$$

2.7 CRITICAL BUCKLING LOADS.

In the present study the critical buckling load parameters of thin rectangular generally orthotropic plates are found by the equilibrium method. An approximate solution to the differential equation 2.32 is obtained by the principle of minimum potential energy using the Raleigh Ritz procedure. Characteristic beam functions are used as admissible functions to represent the deflection of the plate. The analogy shown in the previous section between the vibration and buckling problems can be used to find the buckling loads from the frequency data when the plate has all edges simply supported. For arbitrary boundary conditions, however, the eigenvalue problem is to be solved separately.

i) UNI-AXIAL BUCKLING

For a plate with arbitrary boundary conditions at the edges subjected to inplane compressive load parallel to x-axis only, we can put $\lambda = N_2 = N_{12} = 0$ in equations 2,30. They can then be put in the matrix form

$$[K] \{C\} - x_0 [G] \{C\} = 0 \quad \dots 3.34$$

where $x_0 = N_1 a^2 / D_{22}$

$[K]$ and $[G]$ are real symmetric matrices.

Premultiplying by $[C]^{-1}$, we get

$$[A]\{C\} - x_0 [I]\{C\} = 0 \text{ where } [A] = [C]^{-1}[K]$$

A is a real non-symmetric matrix and its eigenvalues represent the critical buckling loads for different modes of deformation of the plate. Generally, the lowest critical buckling load is of interest and is determined by finding the largest eigenvalue of $[A]^{-1} = [K]^{-1} [G]$.

ii) BI-AXIAL BUCKLING.

The minimum compressive load parallel to x-axis to cause buckling of a plate subjected to another inplane load parallel to y-axis (on the other two edges) can be determined by a procedure similar to the one used in (i). Suitable numerical values are substituted for N_2 in equations 2,30 and the eigenvalue problem is solved exactly in the same manner.

iii) SHEAR BUCKLING.

The minimum critical shear loads along the edges of the

plate to cause buckling of their rectangular generally orthotropic plates can be determined by putting $\lambda = \lambda_1 = \lambda_2 = 0$ in equations 2.30 which can then be put in the form

$$[K] \{c\} = X_0 [G'] \{c\} \quad \dots 3.36$$

where $X_0 = N_{12} a^2 / D_{22}^1$

The matrices $[K]$ and $[G']$ are real and symmetric and the eigenvalue problem can be solved exactly in the same manner as in (i) and (ii).

CHAPTER - III

NUMERICAL RESULTS AND DISCUSSION

3.1 FREQUENCY OF TRANSVERSE VIBRATION.

1) PLATE SIMPLY SUPPORTED AT ALL EDGES: The natural frequencies of transverse vibration of a thin rectangular generally orthotropic plate are calculated by solving the Eigenvalue problem represented by equation 2.20. Numerical results are obtained for a maple plywood plate for which the elastic properties are $D_{11} / D_{126} = 1.643$ and $D_{22} / D_{126} = 4.813$. The rigidities D_{11} , D_{22} , D_{12} , D_{16} and D_{26} are calculated from equations A.9 and 2.7. The elements of the matrix K are evaluated from equations 2.19 taking $p = q = 4$ for different side ratios (a/b) and angles of orthotropicity (θ). For the specially orthotropic case (i.e., $\theta = 0^\circ$ or 90°) K is a 16×16 diagonal matrix and for the generally orthotropic plate (for other values of θ) the orthotropic coupling terms containing D_{16} and D_{26} contribute the off-diagonal terms. However, the diagonal terms are large when compared to others and the matrix is real and symmetric. The eigenvalues of K are evaluated by the Jacobi method. The non-dimensional frequency parameter $\lambda = (P_h \omega^2 c^2 b^2 / \pi^4 D_{11})^{1/2}$ is calculated for the first sixteen nodes of vibration. These nodes are characterised by m and n (in equations 2.19) which represent the number of half waves into which the plate bends along the x and y directions respectively. The frequencies of the first six nodes of vibration are given in table 4 for various side ratios and angles of orthotropicity.

The results for the particular cases when $\theta = 0^\circ$ and 90° are compared with those obtained from the closed form expression derived by Hearmon (5) using the Raleigh's method in Table 3. This method gives the exact frequency for the simply supported case. The comparison shows that the results of the present investigation agree exactly (upto three decimal places) with those of Hearmon. The Raleigh's method, however, cannot be applied for other angles of orthotropicity because of the terms D_{16} and D_{25} in equation 2.13. The effect of the side ratio on the frequency of the two modes ($m = 1, n = 1$ and $m = 1, n = 2$) is shown in figure 2. The frequency decreases rapidly with the increase of the side ratio from a high value, attains a minimum at a particular a/b ratio and increases at a less rapid rate. The value of the side ratio at which the frequency is minimum depends upon the mode shape the angle of orthotropicity and the elastic properties of the plate. Figure 3 shows the variation of the frequency of the first three modes ($m = 1, n = 1, m = 1, n = 2$ and $m = 2, n = 1$) with the angle of orthotropicity for side ratios of 0.4, 1.0, 2.0 and 3.0. For certain modes the frequency increases with the angle of orthotropicity while for others it decreases. There are some modes for which the maximum frequency occurs at a value near about 45° .

The frequency of transverse vibration of a plate subjected to normal inplane load along a pair of opposite edges is calculated from equation 2.20. Numerical values (both tensile and compressive) are substituted for N_1 in equations 2.19 keeping $N_2 = N_{22} = 0$ in them to evaluate the elements of the matrix

K , which is real and symmetric. The eigenvalue problem is solved and the frequency parameters are determined and given in table 5 for various side ratios, angles of orthotropicity and edge loads. Figure 4 shows the effect of the inplane load on the frequency of transverse vibration. The square of frequency decreases linearly with the decrease of the tensile load (or increase of compressive load) and the inplane load corresponding to zero frequency is the static buckling loads of the plate.

ii) PLATE WITH ARBITRARY BOUNDARY CONDITIONS.

The frequencies of transverse vibration of thin rectangular plate with arbitrary boundary conditions at the edges are found by solving the eigenvalue problem represented by equation 2.31. The numerical values of the integrals required in equation are determined by the integration of the beam characteristic functions. These characteristic functions which represent the mode shapes of long, slender beams subjected to transverse vibrations for different boundary conditions are given in table 1.6. n represents the mode number (the number of half waves into which the beam bends). A numerical integration procedure using the "Romberg Integration" is utilized to evaluate all the required integrals. The numerical results are presented in table 2 (a, b and c). The integrals $\int g_m^1 g_n^1 dx$ and $\int g_m^{11} g_n^1 dx$ were not previously reported in the literature whereas the integrals $\int g_m^1 g_n^1 dx$ and $\int g_m^{11} g_n^{11} dx$ were tabulated by Young (3) for clamped-clamped, clamped-free and

free-free beams. The results given in Table 2, for these cases are in close agreement with those of references (3). This numerical data is now substituted in equations 2.20, to evaluate the elements of the matrix K . This matrix is real and symmetric irrespective of the boundary conditions of the plate. The eigenvalues are determined as in case (1) and the ^{non-}dimensional frequency parameters ($\lambda^2, Ph w^2 a^4/D_{22}^1$) are evaluated for the first 16 modes of vibration. A general computer program is made (given in Appendix B) to find the frequencies of vibration of thin rectangular plates with any arbitrary boundary conditions at the edges, with side ratios ranging from 0.5 to 3.5 (in steps of 0.5) and with the angle of orthotropicity ranging from 0° to 90° (in steps of 15°).

(Continued....)

Numerical values of these frequencies for a maple plywood plate ($B_{11} / B_{22} = 3.12$, $B_{12} / B_{22} = 0.1206$ and $B_{33} / B_{22} = .2697$) are evaluated and presented in tables 6a - 6e for the following boundary conditions;

- a) all edges simply supported
- b) all edges clamped
- c) two opposite edges clamped and the others simply supported
- d) one edge clamped and the rest free (cantilever plate)

and e) one edge free and the rest simply supported.

While the frequencies of generally orthotropic plates with arbitrary boundary conditions are calculated for the first time, the specially orthotropic case was solved by Hearmon (8) using the Raleigh's method for thin rectangular plates having various combinations of clamped and supported edges. The fundamental frequencies found from equation 2.31 are compared with those determined from the closed form expressions derived by Hearmon for specially orthotropic case i.e. when $\theta = 0^\circ$ and $\theta = 90^\circ$.

The Raleigh's method is known to give exact frequency for the case of a plate simply supported at all edges, whereas, for other boundary conditions it gives an upper bound for the frequencies. Comparison in table 7 shows a very good convergence of the Raleigh Ritz method to these true values for the ssse plate from the higher side (the difference being of the order of 0.05%). For plates with other boundary conditions, for which comparison could be made, it can be seen that the frequencies found in the

present investigation are less than (of the order of 0.25) those given by Raleigh's method. This indicates that the Ritz's modification of the Raleigh's method improves the convergence to the true value from the higher side considerably. In table 8 the first five frequencies of cantilever plate obtained from equation are compared with those given by Somayajulu and Srinivasan (9) for the case when $\theta = 0$. The values in reference 9 appear to have been wrongly tabulated - the fundamental frequency for side ratios 0.5 and 2.0 having been omitted. Comparison is made after the necessary correction and values agree very closely. This is to be expected since the equations 2.30 become exactly same as those used in reference 9 for angles of orthotropicity of 0 or 90° . The a/b ratio has practically no effect on the fundamental frequency of vibration of a specially orthotropic cantilever plate. Burton (22) reports a similar behaviour of a rectangular isotropic plate.

The fundamental frequency of vibration is plotted against the angle of orthotropicity in figure 6 and 6 for side ratios (a/b) 0.5, 1:0, 2:0 and 3:0 and various boundary conditions. The frequency of clamped - clamped plate reduces with the increase of the angle of orthotropicity, e for a side ratio of 0.5 whereas for other ratios it decreases. The eeee plate has the highest frequency and the cantilever plate the lowest. The frequencies of eeee and secc are wide apart for low a/b ratios and they come closer as this ratio is increased.

3.2 CRITICAL BUCKLING LOADS.

1) PLATE SIMPLY SUPPORTED ON ALL EDGES:

The critical buckling loads of generally orthotropic plates, when all edges are simply supported, can be determined from equation 2.33 by solving the corresponding eigenvalue problem.

a) Uniaxial buckling: For buckling under uniform compressive inplane loads on a pair of opposite edges the eigenvalue problem is exactly same in 3.1 (1). The elements of the matrix K are found from equations 2.19 by putting $\tau = R_2 = R_{12} = 0$. The matrix K is found to be real and symmetric and the eigenvalues are determined using the Jacobi's method. The non-dimensional critical buckling load parameters ($K_c = b^2 E_1 / h^3 E_L$) are evaluated for mahogany plywood plate for which $D_{11}/D_{22} = 3.04$, $D_{126}/D_{22} = 0.498$ and $E_L = 1.35 \times 10^6$ psi. These values for the first three modes of deformation of the plate for various angles of orthotropicity and side ratios are given in table 9. Green and Hearmon (16) have, by using a Fourier Series solution, found the critical buckling loads of generally orthotropic plates and have presented numerical results for a square mahogany plywood plate. They have made approximations by limiting the number of terms in the series to six and simplifying the results to derive closed form expressions for the buckling loads. It is remarked that the results are reasonably accurate for a side ratio of 1 and the accuracy decreases with the increase of the side ratio and the magnitude of the orthotropic coupling terms.

(terms containing D_{16} or D_{25}). The present investigation takes 16 term series for the deflection function and is expected to give better results. These results are compared with those of Green and Hearmon in table 9 for a square plate and are found to be in close agreement. In figure 7 the minimum critical buckling load is plotted against the a/b ratio for various angles of orthotropicity. The behaviour is similar to that of specially orthotropic plates as reported by Das (17). For low values of side ratios the first mode gives the minimum critical buckling load and as a/b ratio increases beyond certain value (indicated by a kink in the curve) the second mode give the minimum buckling load. The angle of orthotropicity alters the minimum buckling loads and the location of the kinks along the curve. For certain a/b ratios the advantage in having a value of θ other than 0° or 90° for increasing the critical buckling load is clearly seen in figure 7.

b) Bi-axial Buckling: The minimum critical normal load parallel to y -axis on two opposite edges required to cause buckling of a thin rectangular generally orthotropic plate, when it is subjected to uniform normal inplane load parallel to the x -axis on the remaining two edges, is determined from equation 2.20. The elements of the matrix K are determined from equations 2.19 by giving suitable values to R_2 keeping $1 + R_{12} \neq 0$. The matrix K is real and symmetric and the eigenvalues are determined by the Jacobi's method. The numeri-

results for mahogany plywood plate ($D_{11} / D_{22} \approx 3.04$ and $D_{126} / D_{22} \approx 0.438$) are given in table 9. The critical buckling loads for the first three modes of deformation are plotted against the inplane load parallel to the y-axis in figure 4. The buckling load (along y-direction) is found to vary linearly with the uniform inplane load acting parallel to the y-axis. A plate loaded in tension requires a higher compressive load to cause buckling of the plate than that required by an unloaded plate. A compressive load, on the other hand, requires a smaller load to buckle the plate. Timoshenko (13) describes a similar behaviour of isotropic plates under uniform edge loads in two perpendicular directions. The lines in figure 4 corresponding to the modes $n = 1$, $n = 2$ and $n = 2$, $n = 1$ are not symmetrical with reference to the R_1 and R_2 axes whereas for the isometric case such a symmetry exists for these and other similar modes.

c) Buckling Under Shear Loads: The minimum shear loads along the edges of a thin rectangular plate required to cause buckling can be determined from equations 2.19 by putting $Z = R_1 = R_2 = 0$. The lowest value of R_{12} which makes the determinant of the coefficient matrix of equations 2.19 equal to zero is the minimum critical buckling load. Starting from a value of zero for R_{12} , it is increased gradually in small steps and the sign of the determinant is found, when the sign changes the step size is reduced and the process is repeated until a value of R_{12} is found to sufficient accuracy, which makes the determinant zero very nearly. This value is the shear buckling

load. The numerical results obtained for the mahogany plywood plate are shown in table 11. The results are compared in table 10 with those of Green and Bearmon (16) for the case when $a/b = 1.0$ and are found to be in close agreement. Figure 9 shows the variation of the shear buckling loads (+ve and -ve) with the angle of orthotropicity.

11) PLATE WITH ARBITRARY BOUNDARY CONDITIONS.

For a plate subjected to arbitrary boundary conditions at the edges the minimum critical buckling load is found by solving the eigenvalue problem represented by equation 3.34. This is a general form of the eigenvalue problem. The $\pi\pi$ minimum eigenvalue of the matrix $[A] = [K]^{-1} [G]$ gives the minimum buckling load. This is determined by taking the inverse of the largest eigenvalue of the inverted matrix ($[A]^{-1} = [G]^{-1} [K]$). A general computer program is made to evaluate the elements of matrices K , G and A^{-1} and to determine the largest eigenvalue of A^{-1} by an iterative procedure. The numerical values of the minimum buckling loads found by this procedure are given in Table 12 for various boundary conditions.

The variation of the minimum buckling load with the angle of orthotropicity is plotted in figure 9.

3.3 CONCLUSIONS.

The frequency of transverse vibration and the critical buckling loads of thin rectangular generally orthotropic plates were evaluated using the Raleigh Ritz method and the principle of

minimum potential energy. The deflections were assumed to be small and the effects of rotating inertia and shear deformation were neglected. In the vibration problem the frequencies are the eigenvalues of a real symmetric matrix. The natural frequencies of vibration for the first sixteen modes were calculated by using the Jacobi's method on the high speed electronic computer for various boundary conditions. But in the buckling problem the critical buckling loads were the eigenvalues of a real non-symmetric matrix (except when the pair of loaded edges are simply supported, in which case it is real and symmetric). The lowest critical buckling load is determined by an iterative scheme.

The effect of the angle of orthotropicity on the vibration and critical buckling loads is shown in the several tables and graphs presented. In some cases an orientation of orthotropicity other than 0° or 90° increases the minimum critical buckling load by as high as 100%. The advantage gained however will depend upon the side ratio, the boundary conditions at the edges and the orthotropic properties of the material. In applications where minimum weight is an important design consideration an investigation of this type is worthwhile in view of the improved buckling characteristics at certain angles of orthotropicity.

3.4 SCOPE OF FURTHER WORK.

The accuracy of the results of higher modes of vibration

could be improved by taking into account the effect of shear deformation and rotating inertia. The analysis could then be applied to thick plates also. Since plate like structural elements, of shapes other than rectangular, are frequently used in air craft and missile applications, it is profitable to extend the present analysis to such shapes (skew, triangular etc.). The effects of large deformations on the vibration characteristic could also be studied for the generally orthotropic plates. Further, the method of the present work could be extended to cylindrical and spherical shells to investigate the effects of angular orthotropicity on the vibration and buckling characteristics.

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APPENDIX 'A'

ELASTIC CONSTANTS FOR ORTHOTROPIC THIN PLATE

A.1 THE GENERALISED HOOKE'S LAW:

Robert Hooke in 1678 proposed his famous law, which in tensor notation can be expressed as

$$T_{ij} = c_{ijkl} S_{kl}$$

and $S_{ij} = s_{ijkl} T_{kl} \quad (i, j, k, l = 1, 2, 3) \quad \text{--- A.1}$

T_{ij} and S_{kl} are components of stress and strain and c_{ijkl} and s_{ijkl} are the elastic stiffnesses and compliances respectively. There are, in general, 81 constants in each of equations A.1 but because of the symmetry relations existing between shear stresses and strains ($T_{ij} = T_{ji}$ and $S_{ij} = S_{ji}$), this number reduces to 26. Bearman (5) shows by a thermodynamic argument that $c_{ijkl} = c_{klij}$ and $s_{ijkl} = s_{klij}$. Thus the number of independent constants in each of the above equations reduces to 21 and the Generalised Hooke's law can be written as

$$T_q = b_{qr} S_p$$

... A.2

and $S_q = s_{qr} T_p \quad (q, r = 1, 2 \dots 6)$

The first equation in the expanded form becomes

$$T_1 = b_{11} S_1 + b_{12} S_2 + b_{13} S_3 + b_{14} S_4 + b_{15} S_5 + b_{16} S_6$$

$$T_2 = b_{21} S_1 + b_{22} S_2 + b_{23} S_3 + b_{24} S_4 + b_{25} S_5 + b_{26} S_6$$

$$T_3 = b_{31}S_1 + b_{32}S_2 + b_{33}S_3 + b_{34}S_4 + b_{35}S_5 + b_{36}S_6$$

$$T_4 = b_{41}S_1 + b_{42}S_2 + b_{43}S_3 + b_{44}S_4 + b_{45}S_5 + b_{46}S_6$$

$$T_5 = b_{51}S_1 + b_{52}S_2 + b_{53}S_3 + b_{54}S_4 + b_{55}S_5 + b_{56}S_6$$

$$T_6 = b_{61}S_1 + b_{62}S_2 + b_{63}S_3 + b_{64}S_4 + b_{65}S_5 + b_{66}S_6$$

A-3

There are thus 21 independent constants in equations A-3, because of the relation $b_{ij} = b_{ji}$. The stiffnesses and compliances relate the components of one second order tensor to those of another (equation A-1) and therefore form a fourth order tensor. They transform from one set of coordinate axes to another according the laws of coordinate transformation. Thus there are 81 transformation equations in 81 constants in the most general case which are reduced to 21 equations in 21 constants due to the relations cited above (5).

A.2 THIN ORTHOTROPIC PLATE.

If a plate is orthotropic, i.e., it possesses three mutually perpendicular planes of elastic symmetry, and one such plane coincides with the plane of the plate, Sokolnikoff (20) has shown that the number of independent elastic constants reduces to 9 and the stress-strain relations become

$$T_1 = b_{11}S_1 + b_{12}S_2 + b_{13}S_3 + 0 + 0 + b_{16}S_6$$

$$T_2 = b_{12}S_1 + b_{22}S_2 + b_{23}S_3 + 0 + 0 + b_{26}S_6$$

$$T_3 = b_{13}S_1 + b_{23}S_2 + b_{33}S_3 + 0 + 0 + b_{36}S_6$$

A-4

$$T_4 = 0 + 0 + 0 + b_{44}S_4 + b_{45}S_5 + 0$$

$$T_5 = 0 + 0 + 0 + b_{54}S_4 + b_{55}S_5 + 0$$

$$T_6 = b_{16}S_1 + b_{26}S_2 + b_{36}S_3 + 0 + 0 + b_{66}S_6$$

In the case of thin plate, i.e. a plate whose lateral dimensions are large in comparison to its thickness, the state of stress is approximately plane and it can be assumed that $T_3 = T_4 = T_5 = 0$. Such a plate is known as a "generally orthotropic" plate and the Hooke's law in its case can be expressed as

$$T_1 = b_{11}S_1 + b_{12}S_2 + b_{16}S_6$$

$$T_2 = b_{12}S_1 + b_{22}S_2 + b_{26}S_6 \quad A.5$$

$$T_6 = b_{16}S_1 + b_{26}S_2 + b_{66}S_6$$

If the two axes of elastic symmetry lying in the plane of the plate are parallel to the x,y coordinate axes, then the plate is known as "specially orthotropic". Then $b_{16} = b_{26} = 0$ and the Hooke's law becomes

$$T_1 = b_{11}S_1 + b_{12}S_2$$

$$T_2 = b_{12}S_1 + b_{22}S_2$$

$$T_6 = b_{66}S_6$$

A.6

It can easily seen that

$$b_{11} = E_1/\mu; \quad b_{12} = E_1 \rightarrow 21/\mu_F \quad b_{22} \rightarrow 22/\mu$$

$$b_{22} = E_2/\mu, \quad b_{66} = E_{12} \quad \text{where } \mu = 1 \rightarrow 12 \rightarrow 21$$

... A.7

If the plate is isotropic we have $E_1 = E_2 = E$,
 $\gamma_{12} = \gamma_{21} = \gamma$ and $\mu = 1 - \gamma^2$, then

$$b_{11} = b_{22} = \frac{E}{(1-\gamma^2)}, \quad b_{12} = \frac{\gamma E}{1-\gamma^2}$$

and $b_{66} = 0$

... A.8

the number of independent constants are then only two since a relation exists between E and G .

A.3 COORDINATE TRANSFORMATION.

For an orthotropic thin plate the elastic constants in any arbitrary directions inclined at an angle θ to the axes of elastic symmetry in the plane of the plate are given by coordinate transformation (5)

$$b_{11} = b'_{11} n^4 + b'_{22} m^4 + 2(b'_{12} + 2b'_{33}) n^2 m^2$$

$$b_{22} = b'_{11} n^4 + b'_{22} m^4 + 2(b'_{12} + 2b'_{33}) n^2 m^2$$

$$b_{12} = (b'_{11} + b'_{22} - 4b'_{66}) n^2 m^2 + b'_{12} (n^4 + m^4)$$

$$b_{16} = [b'_{22} n^2 - b'_{11} m^2 + (b'_{12} + 2b'_{33}) (n^2 - m^2)] \quad \text{mm}$$

$$b_{26} = [b'_{11} n^2 - b'_{22} m^2 + (b'_{12} + 2b'_{33}) (n^2 - m^2)] \quad \text{mm}$$

$$b_{66} = (b'_{11} + b'_{22} + 2b'_{12}) n^2 m^2 + b'_{66} (n^2 - m^2)^2$$

A.9

where b_{11} , b_{22} etc. are elastic constants along arbitrary directions

b'_{11} , b'_{22} etc. are elastic constants along axes of elastic symmetry $n = \cos \theta$ and $m = \sin \theta$,

APPENDIX 'B'

COMPUTER PROGRAM

FREQUENCIES OF GENERALLY ANISOTROPIC PLATES

THIS PROGRAM CALCULATES THE ELEMENTS OF THE 16 X 16 MATRIX K, FOR DIFFERENT SIDE RATIOS AND ANGLES OF ANISOTROPICITY. THE EIGENVALUES ARE DETERMINED BY JACOBI'S METHOD AND THE NON-DIMENSIONAL FREQUENCY PARAMETERS ARE PRINTED FOR THE FIRST 16 MODES OF VIBRATION AS OUT PUT.

```
DIMENSION FA(4,4), FB(4,4), FC(4,4), FD(4,4), FE(4,4), FF(4,4)
DIMENSION BUCK(4,4), A(4,4), D(16,16), EGEM(16), ALAM(16)
DIMENSION LFO F2(4,4), LF1F2(4,4), LF1FO(4,4), Z(16,16), B(6)
DIMENSION x(6), LF1F1(4,4), LF2F1(4,4), LF2FO(4,4), LF0F1
        (4,4)
        BF1FO(4,4), BF1F1(4,4), BF2F1(4,4), BF2FO(4,4), BF1F2(4,4)
        BF0F2(4,4)
```

```
REAL LF0F2, LF1F2, LF1FO, LF1F1, LF2F1, LF2FO, LF0F1
```

READ INTEGRALS OF BEAM CHARACTERISTIC FUNCTIONS AS DATA

```
READ 333, (LF1F1(N, N), N=1,4), N= 1,4
```

```
READ 333, (LF1FO(N, N), N=1,4), N= 1,4
```

```
READ 333, (LF2F1(N,N), N= 1,4), N= 1,4
```

```
READ 333, (LF2FO(N,N), N=1,4), N= 1,4
```

```
READ 333, (BF1FO(N,N), N=1,4), N= 1,4
```

```
READ 333, (BF2F1(N,N), N=1,4) N= 1,4
```

```
READ 333, (BF2FO(N,N), N=1,4) N= 1,4
```

```
READ 334, (X(N), N= 1,4)
```

```
READ 334, (B(N), N= 1,4)
```

C PRINT DATA FOR OUTPUT RECORD

```
PRINT 333, (BF1F1(N,N), N= 1,4), N= 1,4
```

```
PRINT 333, (BF1FO(N,N), N= 1,4), N= 1,4
```

```
PRINT 333, (BF2F1(N,N), N=1,4), N= 1,4
```

PRINT 333, (LF2FO(M,N), M=1,4), M=1,4)
PRINT 333, (BF1FO(M,N), M=1,4), M=1,4)
PRINT 333, (BF2FI(M,N), M=1,4, M= 1,4)
PRINT 333, (BF2FO(M,N), M=1,4), M= 1,4)
PRINT 333, (BF1FI(M,N), M=1,4) M=1,4)
PRINT 334 (B(M), M=1,4)

FORMAT (6F12.3)

33 FORMAT (4F16.8)

34 FORMAT (2 X, 16F8.3)

22 PI=4.*ATAN(1.)

DO 111 M= 1,4

DO 111 M= 1,4

LF0F2 (M,N)= BF2FO(M,N)

LF1F2(M,N)=LF2FI(M,N)

BF1F2(M,N)= BF2FI(M,N)

LF0F1(M,N)= LF1FO(M,N)

BF0F1(M,N)= BF1FO(M,N)

B11, B22 ARE ORTHOTROPIC PROPERTIES OF THE MAPLE 5 PLY-
PLYWOOD PLATE (DATA)

B11=3.32

B22=1.00

B12= 0.1206

B13=0.0

B23=0.0

B33=0.2637

THETA= -15.0

31 THETA,THETA=15.0

T = THETA*PI/180.

C = COS(T)

S = SIN(T)

C2 = C**2

S2 = S**2

C4 = C**4

S4 = S**4

BXY = B12*S.*B33

D11, D22 ARE ORTHOTROPIC PROPERTIES OF THE PLATE FOR
ANGLE OF ORTHOTROPICITY THETA

D11 = B11*C4 + B22*S4 + 2.*BXY*C2*S2

D22 = B22*C4 + B11*S4 + 2.*BXY*C2*S2

D13 = (B22*S2 - B11*C2 + BXY*(C2 - S2))*C*S

D23 = (B22*C2 - B11*S2 - BXY*(C2 - S2))*C*S

D12 = (B11 + B22 - 4.*B33)*C2*S2 + B12*(C4 - S4)

D33 = (B11 + B22 - 2.*B12)*C2*S2 + B33*(C2 - S2)**2

R IS SIDE RATIO = A/B

R = 0.0

R = R=0.5

DO 1 N = 1,4

DO 1 N = 1,4

DO 16 J = 1,4

DO 16 I = 1,4

BN = B(N)

XN = X(N)

BN = B(N)

IF (N.EQ.J) GO TO 11

DEFINI = 0.0

T = THETA = PI/180.

C = COS(T)

S = SIN(T)

C2 = C**2

S2 = S**2

C4 = C**4

S4 = S**4

BXY = B12*S.*B23

C C D11, D22 ARE ORTHOTROPIC PROPERTIES OF THE PLATE FOR ANGLE OF ORTHOTROPICITY THETA

D11 = B11*C4 + B22*S4 + 2.*BXY*C2*S2

B22 = B22*C4 + B11*S4 + 2.*BXY*C2*S2

D13 = (B22*S2 - B11*C2 + BXY*(C2 - S2))*C*S

B23 = (B22*C2 - B11*S2 - BXY*(C2 + S2))*C*S

D12 = (B11 + B22 - 4.*B23)*(C2*S2 + B12*(C4 - S4))

D23 = (B11 + B22 - 2.*B12)*C2*S2 + B23*(C2 - S2)**2

C R IS SIDE RATIO = A/B

R = 0.0

4 R = R*0.5

DO 1 N = 1,4

DO 1 N = 1,4

DO 16 J = 1,4

DO 16 I = 1,4

BN = B(N)

XN = X(N)

BN = B(N)

IF (N.EQ.J) GO TO 21

BNBN = 0.0

T = THETA = PI/180.

C = COS(T)

S = SIN(T)

C2 = C**2

S2 = S**2

C4 = C**4

S4 = S**4

BXY = B12*S.*B23

C DLL, D22 ARE ORTHOTROPIC PROPERTIES OF THE PLATE FOR
ANGLE OF ORTHOTROPICITY THETA

DLL = B11*C4 + B22*S4 + 2.*BXY*C2*S2

D22 = B22*C4 + B11*S4 + 2.*BXY*C2*S2

D12 = (B22*S2 - B11*C2 + BXY*(C2 - S2))*C*S

D23 = (B22*C2 - B11*S2 - BXY*(C2 - S2))*C*S

D13 = (B11 + B22 - 4.*B33)*C2*S2 + B12*(C4 - S4)

D33 = (B11 + B22 - 2.*B12)*C2*S2 + B23*(C2 - S2)**2

C R IS SIDE RATIO = A/B

R = 0.0

4 R = R=0.5

DO 1 M = 1,4

DO 1 N = 1,4

DO 16 J = 1,4

DO 16 I = 1,4

BM = B(N)

XN = X(N)

BN = B(N)

IF (M,NQ,J) GO TO 11

DETERM = 0.0

GO TO 12

20 DEIMI \geq 1.0

12 IF(N, EQ, J) GO TO 11

DELNJ = 0.0

GO TO 13

11 DEMLIJ \geq 1.0

13 DIMNLJ = DEIMI*DELNJ

CMI = LP1FO(N, I)

CIM = LF0F1(N, I)

DMI = LP1F1(N, I)

EMI = LP2F1(N, I)

FMI = LP2FO(N, I)

GIM = LP1F2(N, I)

DIM = DMI

PIM = LF0F2(N, I)

CNJ = BP1FO(N, J)

DNJ = BP1F1(N, J)

ENJ = BP2F1(N, J)

FNJ = BP2FO(N, J)

ENJ = BP1F2(N, J)

CJN = BP0F1(N, J)

DJN = DNJ

PJN = BP0F2(N, J)

PA(I, J) = D11*D11*4*DIMNLJ

PB(I, J) = D12*(PIM*PNJ*PMI*PNJ)*R*R

PC(I, J) = D22*X1*X1*4*R*R*4*DIMNLJ

PD(I, J) = 4.*D33*D11*D11*D11*R*R

PE(I,J) = 2.*D13*(EMI+CJN+ EIM+CJN)*R*R

PE(I,J) = 2.*D23*(CIM+ENJ+ CMI+EJN)*R*R

ABCD = FA(I,J) + FB(I,J) + FC(I,J) + FD(I,J) + PE(I,J) + PF(I,J)

A(I,J) = ABCD

35 CONTINUE

L = 4*I-4+N

J4 = J44

J8 = J+8

J12 = J + 12

C D'S ARE THE ELEMENTS OF THE MATRIX K

D(L,J) = A(I,J)

D(L,J4) = A(2,J)

D(L,J8) = A(3,J)

D(L,J12) = A(4,J)

36 CONTINUE

1 CONTINUE

PRINT 2,R,RX, THETA

2 FORMAT (2X, 3F20.2)

21 FORMAT (2X, 16F8.1)

C ELEMENTS OF MATRIX ARE NORMALIZED

DD = D(1,1)

DO 22 N = 1,16

DO 22 N = 1,16

22 D(N,N) = D(N,N)/DD

CALL JACOBI (16,D,1,MR,3)

DO 999 N=1,16

999 ALAM(N) = SQRT(D(N,N)/PI*4*DD)

PRINT3, (ALAM(N), N = 16)

3 FORMAT (2X, 16F8.3)

3000 CONTINUE

IF(R,LT,3.5) GO TO 4

IF(THETA,LT,90.) GO TO 31

STOP

END

\$IBFTC

JACOBI NODECK

SUBROUTINE JACOBI(N,Q,JVEC, M,V)

C THIS SUB PROGRAM CALCULATES THE N EIGENVALUES,
AND EIGENVECTORS OF A SYMMETRIC MATRIX BY A
METHOD OF DIAGONALISATION.

DIMENSION Q(16,16), V(16,16), X(16), LJ (16)

10 DO 14 I = 1, N

DO 14 J = 1, N

IF (I-J) 12, 11, 12

11 V(I,J) = 1.0

GO TO 14

12 V(I,J) = 0.0

CONTINUE

15 M = 0

17 MI = M-1

DO 30 I = 1, MI

X(I) = 0.0

MJ = I + I

DO 30 J = MJ, N

IF (X(I) = ABS(Q(I,J))) 20,20, 20

20 X(I) = ABS (Q(I,J))

IM (I) = J

CONTINUE

40 DO 70 I = 1, MI

IF (I = 1) 60,60, 45

45 IF (XMAX = X(I)) 60,70,70

60 XMAX = X(I)
 IP = I
 JP = LR (I)
 70 CONTINUE
 EPSI = 1.0E-8
 IF(XMAX - EPSI) 1000, 1000, 148
 148 M = M + 1
 IF (Q(IP,IP) = Q(JP,JP)) 150, 151, 151
 150 TANG = -2.*Q(IP,JP)/(ABS(Q(IP,IP)-Q(JP,JP))
 +SQRT(Q(IP,IP)-
 1 Q(JP,JP))**2 + 4.*Q(IP,JP)**2))
 GO TO 160
 151 TANG = 2.*Q(IP,JP)/(ABS(Q(IP,IP)-Q(JP,JP)) + SQRT
 (Q(IP,IP))
 1 - Q(JP,JP)**2 + 4.*Q(IP,JP)**2))
 160 COSH = 1.0/SQRT(1.0*TANG**2)
 SINE = TANG*COSH
 QII = Q(IP,IP)
 Q(IP,IP) = COSH**2*(QII*TANG*(2.*Q(IP,JP)*TANG + Q
 (JP,JP)))
 Q(JP,JP) = COSH**2*(Q(JP,JP) - TANG*(2.*Q(IP,JP) -
 TANG*QII))
 Q(IP,JP) = 0.0
 IF(Q(IP,IP) = Q(JP,JP)) 152, 153, 158
 152 TEMP = Q(IP,IP)
 Q(IP,IP) = Q(JP,JP)
 Q(JP,JP) = TEMP
 IP(SINE) 154, 155, 155
 154 TEMP = +COSH
 GO TO 170
 155 TEMP = -COSH

170 COSH = ABS(SINE)
 SINE = TEMP
180 DO 350 I = 1,MI
 IF (I = IP) 210, 350, 200
200 IF (I= JP) 210, 350, 210
210 IF(IH(I) = IP) 230,240,230
230 IF(IH(I) = JP) 350,240,350
240 K = IH(I)
250 TEMP = Q(I,K)
 Q(I,K) = 0.0
 MJ = I + 1
 X(I) = 0.0
 DO 320 J = MJ,N
 IF(X(I) = ABS(Q(I,J)))300,300,320
300 X(I) = ABS(Q(I,J))
 IH(I) = J
320 CONTINUE
 Q(I,K) = TEMP
350 CONTINUE
 X(IP) = 0.0
 X(JP) = 0.0
 DO 530 I = I,N
 IF (I = IP) 370,530,420
370 TEMP = Q(I,IP)
 Q(I,IP) = COSH*TEMP + SINE*Q(I,JP)
 IF(X(I)=ABS(Q(I,IP)))380,390,390
380 X(I) = ABS(Q(I,IP))
 IH(I) = IP

390 $Q(I,JP) = \text{SINE} * \text{TEMP} + \text{COSH} * Q(I,JP)$
 $\text{IF}(X(I) = \text{ABS}(Q(I,JP))) 400, 530, 530$
 400 $X(I) = \text{ABS}(Q(I,JP))$
 $\text{IN}(I) = JP$
 GO TO 530
 420 $\text{IP}(I = JP) 430, 530, 480$
 430 $\text{TEMP} = Q(IP,I)$
 $Q(IP,I) = \text{COSH} * \text{TEMP} + \text{SINE} * Q(I,JP)$
 $\text{IF}(X(IP) = \text{ABS}(Q(IP,I))) 440, 450, 450$
 440 $X(IP) = \text{ABS}(Q(IP,I))$
 $\text{IN}(IP) = I$
 450 $Q(I,JP) = -\text{SINE} * \text{TEMP} + \text{COSH} * Q(I,JP)$
 $\text{IF}(X(I) = \text{ABS}(Q(I,JP))) 400, 530, 530$
 480 $\text{TEMP} = Q(IP,I)$
 $Q(IP,I) = \text{COSH} * \text{TEMP} + \text{SINE} * Q(JP,I)$
 $\text{IF}(X(IP) = \text{ABS}(Q(IP,I))) 490, 500, 500$
 490 $X(IP) = \text{ABS}(Q(IP,I))$
 $\text{IN}(IP) = I$
 500 $Q(JP,I) = -\text{SINE} * \text{TEMP} + \text{COSH} * Q(JP,I)$
 $\text{IF}(X(JP) = \text{ABS}(Q(JP,I))) 510, 530, 530$
 530 $X(JP) = \text{ABS}(Q(JP,I))$
 $\text{IN}(JP) = I$
 530 CONTINUE
 $\text{IP}(JVNC) 540, 40, 540$
 540 DO 550 $I = 1, N$
 $\text{TEMP} = V(I,IP)$
 $V(I,IP) = \text{COSH} * \text{TEMP} + \text{SINE} * V(I,JP)$
 550 $V(I,JP) = -\text{SINE} * \text{TEMP} + \text{COSH} * V(I,JP)$
 GO TO 40
 1000 RETURN
 END
 § ENTRY

APPENDIX - C

TABLES

卷之三

CHILOE AND THE ISLANDS

1	1	4,600	2,000	6,700	0,350	2,000	-1,500	0,000	0,000
1	2	-7,300	0,700	1,000	1,074	0,384	0,641	0,000	0,000
1	3	5,600	0,300	1,505	3,304	-0,305	0,030	0,000	0,000
1	4	-9,800	0,100	1,000	5,294	0,200	0,000	0,000	0,000
2	1	-9,300	0,700	-11,705	8,494	-1,105	-22,308	10,149	
2	2	9,417	0,000	65,700	67,351	-2,000	-16,502	-11,512	
2	3	0,000	0,000	15,305	14,397	2,004	30,826	1,394	
2	4	-22,308	0,000	15,305	15,157	-3,702	-10,821	0,592	
3	1	10,502	0,000	10,400	10,400	0,000	-6,116	-7,151	21,025
3	2	0,000	0,000	10,400	10,400	0,000	-201,888	15,077	
3	3	0,000	0,000	10,400	10,400	0,000	-40,808	-62,838	
3	4	0,000	0,000	10,400	10,400	0,000	210,804	5,748	
3	5	0,000	0,000	10,400	10,400	0,000	-60,570	31,805	
3	6	0,000	0,000	10,400	10,400	0,000	-71,800	35,004	
3	7	0,000	0,000	10,400	10,400	0,000	-659,204	18,816	
3	8	0,000	0,000	10,400	10,400	0,000	-104,269	-80,084	

INTEGRALS OF CHARACTERISTIC HARMONIC FUNCTIONS

N	n	FREE - FREE		SUPPORTED - SUPPORTED	
		$\int_0^L \psi_n(x) dx$	$\int_0^L \psi_n'(x) dx$	$\int_0^L \psi_n(x) dx$	$\int_0^L \psi_n'(x) dx$
1	1	0.000	0.000	0.000	0.000
1	2	0.000	0.000	0.000	0.000
1	3	0.000	0.000	0.000	0.000
1	4	0.000	0.000	0.000	0.000
2	1	0.000	-0.000	0.000	-0.000
2	2	12.000	0.000	0.000	0.000
2	3	0.000	0.000	0.000	0.000
2	4	15.000	0.000	0.000	0.000
3	1	0.000	0.000	0.000	0.000
3	2	0.000	-0.000	-0.000	0.000
3	3	40.000	0.000	-0.000	0.000
3	4	0.000	1.014	0.000	0.000
4	1	0.000	-1.000	0.000	-1.007
4	2	10.000	0.000	0.000	0.000
4	3	0.000	-0.000	-0.000	0.000
4	4	100.000	0.000	-0.000	0.000

TABLE 3

Fundamental frequencies $[\lambda = (\rho h^2 a^2 b^2 / \pi^4 D_{126}^{'})^{1/2}]$ of specially orthotropic SSSS plates (Maple plywood) $D_{11}^{'} / D_{126}^{'} = 1.545$, $D_{22}^{'} / D_{126}^{'} = 4.815$

Side ratio a/b	ANGLE OF ORTHOTROPICITY DEGREES			
	0		90	
	From equation 2.20	by Raleigh's Method Harmon (Ref. 5)	From equation 2.20	by Raleigh's Method Harmon (Ref. 5)
0.4	5.825	5.825	5.825	5.825
0.6	2.832	2.832	3.931	3.931
0.8	2.737	2.737	3.242	3.242
1.0	2.891	2.891	2.891	2.891
1.2	3.165	3.165	2.750	2.750
1.4	3.496	3.496	2.735	2.735
1.6	3.835	3.835	2.798	2.798
1.8	4.251	4.251	2.915	2.915
2.0	4.652	4.652	3.032	3.032
2.2	5.031	5.031	3.235	3.235
2.4	5.476	5.476	3.424	3.424
2.6	5.905	5.905	3.625	3.625
2.8	6.391 519	6.319	3.835	3.835
3.0	6.745	6.745	4.052	4.052
3.2	7.172	7.172	4.274	4.274
3.4	7.601	7.601	4.500	4.500
3.6	8.031	8.031	4.730	4.730
3.8	8.462	8.462	4.961	4.961
4.0	8.894	8.894	5.195	5.195

TABLE 4

Natural frequencies [$\lambda = (\rho h^2 a^2 b^2 / \pi^4 D_{123}^2)^{1/2}$] of generally orthotropic SSSS plate (Maple) plywood plate, $D_{11}^2 / D_{123}^2 = 1.545$; $D_{22}^2 / D_{123}^2 = 4.815$

Side Ratio a/b	n	n	ANGLE OF ORTHOTROPICITY DEGREES						
			0	15	30	45	60	75	90
0.4	1	1	5.523	5.689	4.095	4.557	5.047	5.496	5.686
	2	1	12.770	12.810	13.462	15.568	18.440	21.076	22.126
	3	1	28.283	28.085	29.087	33.600	40.735	47.085	49.546
	4	1	50.016	49.422	50.845	59.191	72.278	85.568	89.958
	1	2	5.474	5.379	6.587	6.385	6.755	6.556	6.485
	1	3	9.486	9.762	10.216	10.150	9.461	8.617	8.251
1.0	1	1	2.891	3.022	3.292	3.427	3.292	3.022	2.891
	2	1	6.124	6.452	7.035	8.795	8.398	9.154	9.303
	3	1	12.153	12.707	12.259	16.828	16.490	19.508	20.234
	4	1	20.780	21.036	22.254	26.145	30.798	34.251	35.573
	1	2	9.305	9.154	9.899	7.535	7.035	6.452	6.124
	1	3	20.234	19.506	18.490	16.256	12.259	12.707	12.157
1.6	1	1	5.885	5.840	5.801	5.897	5.408	5.005	2.798
	2	1	5.474	5.755	6.361	6.215	6.656	6.629	6.485
	3	1	9.893	9.549	10.539	11.255	11.745	15.358	15.200
	4	1	14.093	14.641	15.963	20.879	21.745	22.542	22.743
	1	2	14.344	15.780	12.586	10.361	9.596	9.797	6.549
	1	3	51.885	50.468	26.859	22.468	19.564	18.524	18.455

Continued

Side Ratio a/b	n	n	ANGLE OF ORNITHOPICITY DEGREES						
			0	15	30	45	60	75	90
2.0	1	1	4.652	4.549	4.535	4.040	3.673	5.269	5.082
	2	1	5.781	5.975	6.438	6.740	6.609	6.067	5.781
	3	1	8.276	8.721	9.703	10.558	11.249	10.253	11.029
	4	1	12.246	12.916	14.363	16.273	16.051	19.156	18.607
	1	2	17.783	17.005	15.109	12.822	10.381	10.552	10.390
	1	3	39.722	37.783	32.902	27.783	23.474	22.715	22.785
	2	1	6.745	6.460	5.894	5.184	4.470	4.184	4.052
3.0	2	1	7.855	7.813	7.365	7.189	6.782	5.965	5.518
	3	1	9.872	9.951	9.586	10.077	9.918	9.141	8.672
	4	1	10.918	11.635	15.098	14.173	14.822	15.855	15.820
	1	2	20.481	25.183	21.694	18.207	16.826	15.356	15.190
	1	3	50.307	50.397	49.645	50.900	54.456	55.455	55.514
	2	1	9.894	9.493	7.489	6.588	5.618	5.267	5.195
	3	1	9.305	9.070	8.589	8.032	7.372	6.531	6.184
4.0	3	1	10.140	10.170	10.500	10.427	9.964	9.629	9.189
	4	1	11.103	12.122	15.229	15.075	15.470	12.266	11.565
	1	2	25.217	32.459	28.094	25.782	20.555	19.904	20.035
	1	3	70.093	75.016	64.564	52.675	45.409	44.315	45.022

TABLE 8

Buckling Loads and Natural Frequencies of Specially orthotropic
 3638 plates (Maple) plywood plate $D_{11}^{1/2} / D_{120}^{1/2} = 1.545$, $D_{22}^{1/2} / D_{120}^{1/2} = 4.813$)

$\frac{R_1}{\pi^2 D_{11}}$	$\frac{R_2}{\pi^2 D_{11}}$	$\frac{R_2}{\pi^2 D_{11}} \left(\approx \frac{R_2}{\pi^2 D_{11}} \sqrt{\frac{D_{22}}{D_{11}}} \right)$		
		m n	Side Ratio	Side Ratio
			0.5	1.0
-8	1 1	9.092	9.615	15.595
	1 2	2.404	3.848	24.581
	2 1	48.507	49.475	57.888
-4	1 1	8.092	5.615	11.595
	1 2	1.404	2.848	23.581
	2 1	32.507	33.475	41.888
0	1 1	1.092	1.615	7.888
	1 2	0.404	1.848	22.581
	2 1	16.507	17.475	25.888
4	1 1	-0.993	-0.595	5.595
	1 2	-0.593	0.848	21.581
	2 1	0.307	1.475	9.888
8	1 1	-0.993	-0.595	-0.597
	1 2	-1.093	-0.152	20.581
	2 1	-16.505	-14.595	-0.184

Natural Frequency $\omega \lambda = \left(\frac{R_2}{\pi^2 D_{11}} / \frac{D_{22}}{D_{11}} \right)^{1/2}$

TABLE 6a

Frequencies $[\lambda = (ph^2 \omega^4 / \pi^4 D_{22}^2)^{1/2}]$ of generally orthotropic CCCC plates (Maple plywood, $D_{11}^1 / D_{22}^1 = 3.117$, $D_{33}^1 / D_{22}^1 = 4.262$, $D_{12}^1 / D_{22}^1 = 0.12$)

Side ratio	---	ANGLE OF ORTHOTROPICITY DEGREES						-----
		0	15	30	45	60	75	
0.5	1	4.101	5.942	5.522	5.029	2.657	2.579	2.573
	2	4.500	4.407	4.529	4.696	5.775	5.735	5.625
	3	5.627	5.529	5.660	5.520	5.260	5.018	6.188
	4	6.974	7.122	7.590	7.560	6.684	6.457	6.471
	5	11.151	10.617	9.298	8.005	7.901	7.661	7.323
	6	11.462	11.110	10.255	9.180	8.271	8.148	8.184
1.0	1	4.813	4.765	4.808	4.845	4.809	4.765	4.813
	2	7.907	7.905	8.255	8.490	8.255	7.995	7.907
	3	11.580	11.208	10.379	9.877	10.379	11.208	11.580
	4	13.471	13.593	13.258	13.257	13.256	13.593	13.471
	5	15.738	14.077	14.897	15.257	14.897	14.077	15.738
	6	18.228	18.547	18.357	17.262	18.357	18.547	18.228
1.5	1	6.821	6.819	7.008	7.207	8.470	9.021	9.527
	2	12.620	12.518	11.717	11.845	11.908	11.732	11.705
	3	18.470	15.388	18.188	17.818	17.475	18.742	18.470
	4	19.500	18.397	17.980	18.384	21.084	25.975	25.500
	5	23.610	22.820	23.070	23.250	23.248	24.205	25.202
	6	26.500	26.795	26.610	26.097	26.305	26.507	26.700
2.0	1	10.900	10.610	10.740	12.112	14.027	15.769	16.417
	2	15.492	14.910	15.081	16.225	17.514	17.860	18.058
	3	24.765	22.471	21.041	22.070	22.659	22.115	21.707
	4	25.005	25.350	26.735	30.239	30.391	26.729	27.925
	5	30.200	30.644	31.208	32.619	37.134	42.469	44.530
	6	33.550	32.521	33.004	35.659	40.901	44.404	45.000

Side Ratio a/b	Mode No.	ANGLES OF ORTHOTROPICITY DEGREES						
		0	15	30	45	60	75	90
2.5	1	15.137	15.111	15.750	18.002	21.573	24.256	25.369
	2	19.214	19.051	19.925	22.156	24.478	26.103	26.878
	3	27.736	25.656	25.148	27.816	29.615	29.793	29.824
	4	33.882	30.935	35.561	37.157	37.910	35.876	34.805
	5	40.657	39.826	41.338	47.519	57.366	65.969	69.541
	6	42.687	43.879	45.867	53.056	61.143	67.893	70.491
5.0	1	21.214	21.108	21.696	25.253	30.502	34.861	36.548
	2	24.600	24.570	25.912	29.388	35.553	36.353	37.474
	3	32.004	30.935	30.811	35.036	38.356	39.654	39.935
	4	44.111	41.903	41.438	44.594	44.795	44.920	44.288
	5	56.952	56.471	56.511	67.471	62.034	64.744	100.759
	6	69.442	69.172	68.925	73.108	85.320	90.600	102.738
5.5	1	29.422	28.936	29.928	35.795	40.526	45.374	49.341
	2	31.572	31.524	35.934	37.979	43.849	48.864	50.352
	3	37.910	36.495	37.934	45.742	45.805	51.604	52.459
	4	49.000	47.663	48.669	55.355	56.049	56.563	56.220
	5	77.220	75.868	72.565	91.000	111.159	123.751	135.545
	6	79.490	79.059	81.104	96.770	114.930	130.686	136.570

TABLE 6 b

Frequencies $[\lambda = (\rho h v^2 a^4 / \pi^2 D_{22})^{1/2}]$ of generally orthotropic SSSC plate
(Maple plywood $D_{11}^{1/2} / D_{22}^{1/2} \approx 5.117$, $D_{33}/D_{22}^{1/2} \approx 282$; $D_{12}^{1/2} / D_{22}^{1/2} \approx 0.12$)

Side Ratio a/o	Mode No.	ANGLE OF ORTHOTROPICITY (DEGREES)						
		0	15	30	45	60	75	90
0.5	1	1.959	1.948	1.900	1.800	1.869	1.873	1.850
	2	2.657	2.721	3.030	3.111	2.999	3.037	3.181
	3	3.969	4.205	4.619	4.705	4.517	4.378	4.310
	4	5.034	6.141	6.156	5.280	4.834	4.833	5.411
	5	7.138	6.192	6.754	6.978	6.638	6.139	5.769
	6	7.834	7.500	7.587	7.048	6.808	7.012	7.635
1.0	1	3.141	3.229	3.452	3.753	4.025	4.282	4.318
	2	6.945	7.020	7.048	6.910	6.651	6.345	6.200
	3	7.855	7.824	8.134	8.910	9.353	10.744	10.522
	4	10.829	10.900	11.545	11.451	11.175	10.936	10.552
	5	12.891	13.000	13.941	15.851	15.087	12.784	12.726
	6	15.877	15.857	14.972	16.200	16.814	16.358	16.029
1.5	1	5.725	5.208	6.250	7.079	8.105	8.941	9.288
	2	9.501	9.451	9.745	10.508	10.640	10.881	10.567
	3	14.642	14.750	15.195	15.556	15.141	14.392	13.951
	4	17.595	18.050	18.651	17.957	21.148	20.516	19.983
	5	17.626	19.054	20.700	22.580	22.036	24.050	25.127
	6	25.914	25.229	25.142	25.095	24.933	26.699	26.215
2.0	1	9.501	9.645	10.814	11.750	13.517	15.372	16.248
	2	12.968	12.698	15.528	15.026	15.824	17.653	17.272
	3	19.807	18.768	18.648	20.032	20.682	20.257	19.995
	4	25.555	25.410	26.517	26.122	27.410	26.899	24.901
	5	27.771	28.005	28.154	30.058	30.358	32.342	44.454
	6	31.559	29.957	31.542	35.510	40.188	45.957	45.409

Side Ratio a/b	Mode No.	ANGLE OF ONSET OF DIPOLY						
		0	15	30	45	60	75	90
		14.627	14.621	15.525	17.696	21.153	24.105	25.246
2.5	1	17.055	17.303	18.505	21.003	23.000	25.470	26.127
	2	25.315	22.557	25.403	26.159	27.010	28.510	28.247
	3	34.058	32.692	32.903	34.521	34.644	33.135	32.205
	4	39.578	39.334	40.773	47.166	57.170	65.601	69.255
	5	41.547	42.404	45.603	52.034	60.469	67.407	70.185
	6	20.850	20.728	21.589	24.268	30.111	34.538	36.252
3.0	1	22.094	23.195	24.657	26.452	32.820	35.868	37.051
	2	28.286	27.950	29.548	33.618	36.872	38.442	38.982
	3	38.014	37.615	39.148	42.144	45.862	42.766	42.266
	4	56.748	56.186	57.928	67.169	81.851	94.647	100.507
	5	58.568	59.152	62.818	72.189	88.200	98.243	102.188
	6	26.179	27.959	29.849	35.557	40.694	46.888	49.862
3.5	1	30.001	30.295	32.204	37.118	45.217	49.140	50.011
	2	54.857	54.618	56.977	42.421	47.450	50.575	51.622
	3	48.404	48.715	48.551	51.050	55.849	54.598	54.568
	4	77.050	76.131	78.255	90.809	111.012	128.840	135.457
	5	78.775	79.025	83.153	95.804	114.595	150.251	153.559

TABLE 6a

Frequencies $\lambda = \left(\frac{\rho h^2 a^4}{\pi^4 D_{22}^2} \right)^{1/2}$ of generally orthotropic SSSS plate
(simple plywood, $D_{11}'/D_{22}' = 3.117$, $D_{33}'/D_{22}' = .232$, $D_{12}'/D_{22}' = 0.12$)

Side Ratio a/b	Mode No.	ANGLE OF ORTHOTROPICITY DEGREES						
		0	15	30	45	60	75	90
0.5	1	1.873	1.842	1.745	1.581	1.579	1.560	1.532
	2	2.327	2.454	2.600	2.590	2.547	2.550	2.527
	3	3.831	3.554	3.911	3.859	3.780	3.009	4.162
	4	4.350	5.225	5.824	5.161	4.485	4.217	4.440
	5	7.161	6.803	6.109	6.350	5.895	5.259	4.950
	6	7.490	7.000	7.000	6.592	6.092	6.381	6.592
1.0	1	2.327	2.558	2.472	2.555	2.477	2.357	2.527
	2	4.350	5.018	5.291	5.659	5.659	5.176	4.880
	3	7.490	7.361	7.075	6.855	6.841	7.252	7.490
	4	9.309	9.087	9.210	9.051	9.008	9.009	9.309
	5	9.797	10.309	11.126	12.047	11.126	10.070	9.797
	6	15.326	15.502	15.909	15.160	14.057	14.116	15.326
1.5	1	3.831	3.500	3.725	4.145	4.372	4.435	4.440
	2	8.164	7.549	7.291	7.624	7.740	7.020	6.592
	3	9.797	10.457	10.357	12.365	12.909	11.761	11.092
	4	15.326	12.401	11.709	12.775	14.136	15.378	14.291
	5	16.853	12.057	17.684	18.004	17.937	17.371	17.750
	6	20.945	19.081	18.357	20.542	20.205	18.604	17.945
2.0	1	4.350	5.008	5.509	6.320	6.565	7.338	7.490
	2	9.309	9.737	9.302	10.299	10.859	9.737	9.309
	3	15.729	14.457	15.201	15.512	15.804	14.350	15.592
	4	18.750	16.773	17.005	20.420	21.559	21.041	19.730
	5	19.790	21.941	22.905	24.576	24.427	27.027	28.644
	6	26.359	25.009	24.455	26.504	26.345	26.571	26.661

Side ratio	Mode No.	ANGLES OF ORTHOTROPICITY DEGREES						
		0	15	30	45	60	75	90
2.5	1	7.091	7.189	7.044	9.030	10.287	11.141	11.644
	2	11.017	10.547	11.210	15.374	14.034	15.473	15.019
	3	19.037	15.922	15.403	19.059	19.810	17.808	16.807
	4	25.701	24.691	24.710	28.151	27.727	24.553	22.527
	5	28.864	27.235	28.536	31.601	37.204	42.507	44.535
	6	31.103	30.167	35.453	37.345	41.487	44.658	45.778
3.0	1	9.037	9.055	10.657	12.265	14.299	15.750	16.201
	2	15.026	15.070	14.249	17.022	18.184	18.071	17.759
	3	20.045	17.932	18.555	22.048	24.002	22.194	20.945
	4	32.655	28.909	29.413	35.094	38.429	28.517	26.370
	5	36.868	35.913	38.509	44.191	55.120	60.951	65.932
	6	39.149	40.700	44.560	50.411	57.450	65.051	65.162
3.5	1	15.002	15.040	15.080	15.226	15.905	21.105	22.024
	2	18.032	18.025	17.001	21.011	25.020	25.405	25.405
	3	25.050	20.730	22.074	27.429	29.026	27.471	26.507
	4	34.002	32.022	35.003	38.520	37.705	35.808	31.277
	5	49.075	49.400	51.355	50.505	71.903	68.897	66.925
	6	52.000	55.055	57.050	65.707	70.197	64.814	60.097
4.0	1	18.729	18.745	17.780	20.455	24.404	27.475	28.644
	2	19.720	19.050	22.026	25.020	28.538	29.078	29.931
	3	28.059	24.026	26.449	32.503	34.788	35.827	32.655
	4	37.235	35.003	38.777	45.005	45.700	50.505	37.235
	5	64.000	64.155	66.501	75.700	95.500	107.000	115.417
	6	66.016	67.001	72.000	85.500	97.005	100.005	114.574

TABLE 64

Frequencies $[\lambda = (\pi^2 a^4 / \pi^4 D_{11})^{1/2}]$ of generally orthotropic SSW plate
(Maple plywood).

Side ratio a/b	Mode No.	ANGLE OF ORTHOTROPICITY						
		0	15	30	45	60	75	
0.5	1	1.789	1.710	1.597	1.271	1.105	1.045	1.038
	2	1.965	2.020	2.087	2.035	1.819	1.535	1.402
	3	2.622	2.635	3.169	4.096	3.059	2.833	2.837
	4	3.789	4.107	4.692	5.269	4.101	3.993	4.041
	5	7.005	6.741	6.049	5.135	4.934	4.583	4.584
	6	7.279	7.064	6.499	5.794	5.114	4.894	5.192
1.0	1	1.852	1.790	1.654	1.518	1.370	1.214	1.145
	2	2.655	2.639	3.211	5.447	3.591	2.945	2.606
	3	6.113	6.215	6.927	6.170	4.673	4.183	4.152
	4	7.158	6.914	6.851	7.040	6.918	6.182	5.907
	5	7.941	8.038	8.409	8.905	8.671	8.105	8.159
	6	10.490	10.735	11.129	10.837	9.550	9.330	9.487
1.5	1	1.954	1.881	1.755	1.608	1.422	1.255	1.107
	2	3.874	3.827	3.933	4.569	4.032	4.475	4.535
	3	7.232	7.017	6.999	6.488	5.655	5.121	4.793
	4	9.031	8.622	8.194	8.299	8.509	8.717	7.701
	5	12.543	12.701	13.157	12.803	10.894	9.430	9.545
	6	16.040	14.935	15.757	14.694	15.513	14.050	12.616
2.0	1	2.021	1.952	1.892	1.774	1.618	1.470	1.303
	2	5.556	4.555	3.695	4.761	5.288	4.872	4.562
	3	7.408	7.581	6.905	6.562	7.503	6.355	7.787
	4	10.753	8.935	7.905	8.601	10.151	9.842	9.600
	5	16.250	15.688	14.516	15.807	14.391	11.976	10.664
	6	19.469	16.610	16.701	16.379	16.997	16.687	15.539

Side ratio a/b	Mode No.	ANGLE OF ORTHOTROPICITY						
		0	15	30	45	60	75	90
2.5	1	2.255	1.658	-----	2.288	2.533	2.051	1.728
	2	7.727	4.444	1.872	4.971	5.974	5.538	4.833
	3	7.594	7.051	3.809	8.474	10.832	10.836	9.824
	4	12.689	8.711	6.588	10.971	11.789	11.785	11.636
	5	18.439	18.211	14.685	18.595	17.889	15.985	14.495
	6	21.585	18.212	20.159	20.351	18.545	17.874	16.941
3.0	1	2.442	-----	-----	2.359	2.955	2.376	1.937
	2	7.817	2.847	-----	4.905	6.714	5.854	5.229
	3	10.386	5.099	1.470	8.015	11.725	10.974	10.309
	4	15.495	9.987	9.101	15.243	15.656	14.822	13.542
	5	18.669	18.392	16.454	17.681	18.617	17.955	17.340
	6	24.149	20.919	24.094	24.941	25.208	20.921	19.185
3.5	1	2.646	-----	-----	1.700	5.405	2.712	2.215
	2	8.075	-----	-----	4.649	7.499	6.418	5.611
	3	13.532	1.915	-----	7.500	12.861	11.854	10.748
	4	18.938	10.975	9.969	14.981	19.491	18.875	17.904
	5	20.615	17.592	19.075	20.500	20.990	21.510	21.904
	6	27.029	24.104	26.372	26.897	26.404	26.050	24.737

TABLE 6c

Frequencies $[\lambda = (\rho h^2 a^4 / \pi^2 D_{22}^2)^{1/2}]$ of generally orthotropic C77 plate
(Maple plywood, $D_{11}'/D_{22}' \approx 3.117$, $D_{33}'/D_{22}' \approx 4.292$, $D_{12}'/D_{22}' \approx 0.412$)

Side ratio a/b	Node No.	ANGLE OF ORTHOTROPICITY						
		0	15	30	45	60	75	90
0.5	1	0.629	0.590	0.565	0.410	0.355	0.349	0.356
	2	0.731	0.756	0.775	0.726	0.582	0.500	0.515
	3	1.135	1.240	1.365	1.395	1.174	1.188	1.200
	4	2.001	2.124	2.320	2.354	2.196	2.195	2.235
	5	3.945	3.715	3.175	2.697	2.315	2.384	2.496
	6	4.033	3.959	3.618	3.257	2.985	2.693	2.957
1.0	1	0.629	0.563	0.490	0.389	0.350	0.349	0.356
	2	0.990	1.053	1.197	1.220	1.086	0.997	0.795
	3	2.740	2.840	2.951	2.990	2.148	2.178	2.280
	4	3.945	3.923	3.191	3.250	3.240	3.114	2.990
	5	4.402	4.300	4.357	4.394	4.383	4.285	4.231
	6	6.147	6.400	6.970	6.950	6.315	6.105	6.070
1.5	1	0.629	0.490	0.381	0.321	0.344	0.347	0.356
	2	1.135	1.279	1.399	1.537	1.592	1.259	1.031
	3	3.260	3.408	3.672	3.631	2.142	2.178	2.280
	4	4.914	4.890	4.611	4.750	5.000	4.217	3.795
	5	5.523	5.672	5.675	6.125	6.000	6.115	6.245
	6	6.950	6.277	7.021	7.940	6.111	6.349	7.011
2.0	1	0.629	0.475	-----	0.265	0.345	0.348	0.356
	2	1.532	1.231	-----	1.027	1.031	1.010	1.390
	3	3.910	2.916	1.795	2.614	2.326	2.170	2.290
	4	5.537	4.625	2.455	4.007	5.070	5.220	4.474
	5	9.450	8.477	6.004	7.485	6.205	6.159	6.245
	6	11.041	9.479	9.393	9.720	11.017	6.939	6.945

Side Ratio a/b	Mode No.	ANGLE OF ORTHOTROPICITY						
		0	15	30	45	60	75	90
2.5	1	0.829	0.925	0.944	0.956
	2	1.791	1.968	1.953	1.814
	3	3.940	1.892	...	1.810	2.651	2.204	2.250
	4	6.317	5.287	2.197	2.956	6.585	5.880	5.270
	5	11.029	7.716	5.821	7.910	6.288	6.581	6.245
	6	15.517	11.874	11.200	10.565	11.986	11.245	9.999
3.0	1	0.829	0.909	0.943	0.956
	2	2.036	1.891	2.101	1.893
	3	3.940	5.188	2.414	2.250
	4	7.074	3.230	1.492	3.020	5.544	5.954	6.082
	5	11.031	6.685	5.287	6.850	9.680	7.887	6.245
	6	14.489	12.176	12.002	12.688	11.170	11.655	11.325
3.5	1	0.829	0.929	0.940	0.956
	2	2.342	1.782	2.110	2.175
	3	3.940	5.850	2.782	2.250
	4	7.052	1.751	0.811	2.617	5.081	5.922	6.244
	5	11.031	5.926	5.510	5.821	10.577	6.642	6.905
	6	15.552	13.032	14.215	15.756	11.585	11.899	12.257

TABLE 7

Frequencies [$\lambda = (\rho h^2 a^4 / \pi^2 D_{22}^2)^{1/2}$] of specially orthotropic plates (Maple plywood $D_{11}'/D_{22}' = 3.117$; $D_{12}'/D_{22}' = .12$; $D_{66}'/D_{22}' = .262$)

Angle of ortho- tropicity e degrees	Side Ratio a/b	CCCC		SSCC		SSSS	
		From Eqn. 2.31	By Raleigh's Method	From Eqn. 2.31	By Raleigh's Method	From Eqn. 2.31	By Raleigh's Method
0	0.5	4.104	4.114	1.959	1.959	1.873	1.871
	1.0	4.813	4.817	3.141	3.140	2.327	2.325
	1.5	6.821	6.827	5.723	5.722	3.331	3.328
	2.0	10.306	10.321	9.581	9.519	4.930	4.926
	2.5	15.197	15.170	14.627	14.625	7.091	7.087
	3.0	21.214	21.269	20.830	20.828	9.787	9.783
	3.5	28.482	28.564	28.179	28.176	13.002	12.998
90	0.5	2.576	2.580	1.550	1.550	1.232	1.232
	1.0	4.813	4.817	4.318	4.315	2.327	2.325
	1.5	9.527	9.545	9.263	9.257	4.440	4.436
	2.0	16.417	16.457	16.242	16.239	7.490	7.485
	2.5	25.369	25.437	25.246	25.233	11.444	11.437
	3.0	36.348	36.451	36.262	36.236	16.291	16.281
	3.5	49.341	49.484	49.262	49.236	22.024	22.012

TABLE 9

Frequencies [$\lambda = (\rho h^2 a^4 / \pi^4 D_{22}^{'})^{1/2}$] of specially orthotropic cantilever (CFFF) plates (Maple plywood $D_{11}^{'}/D_{22}^{'} = 3.117$; $D_{12}^{'}/D_{22}^{'} = .12$; $D_{66}^{'}/D_{22}^{'} = .26$)

Side ratio a/b	Mode No.	from equation 2.31	from Reference 9
0.5	1	0.629	0.629
	2	0.731	0.730
	3	1.133	1.127
	4	2.001	2.000
	5	3.943	---
1.0	1	0.629	0.629
	2	0.966	0.966
	3	2.740	2.740
	4	3.945	3.942
	5	4.408	---
2.0	1	0.629	0.629
	2	1.239	1.195
	3	3.939	3.938
	4	4.944	---
	5	5.523	5.601

TABLE 9

Buckling loads ($k_b = N_1 b^2 / h^3 E_L$) of generally orthotropic SSSS plates (Mahogany plywood $D'_{11}/D'_{22} = 3.04$, $D'_{120}/D'_{22} = 0.438$, $E_L = 1.35 \times 10^6$ p.s.i.)

Side Ratio n	n	ANGLE OF ORTHOTROPICITY, ϵ							
		0	15	30	45	60	75	90	
0.4	1	1	4.408	4.000	3.431	2.806	2.369	1.833	1.670
	2	1	16.4924	16.414	11.406	8.004	5.991	5.619	5.702
	3	1	37.824	36.886	31.287	16.352	12.321	12.043	12.541
1.0	1	1	1.080	1.209	1.494	1.645	1.494	1.209	1.080
	2	1	2.922	2.828	2.690	2.650	1.688	1.383	1.236
	3	1	6.237	5.764	4.822	3.463	2.284	2.064	2.240
1.6	1	1	1.015	1.200	1.598	1.892	1.966	1.975	1.990
	2	1	1.377	1.468	1.628	1.618	1.384	1.090	1.963
	3	1	2.606	2.669	2.693	1.931	1.621	1.294	1.153
2.0	1	1	1.286	1.420	1.843	2.234	2.519	2.789	2.922
	2	1	1.080	1.233	1.516	1.687	1.406	1.174	1.080
	3	1	1.795	1.783	1.809	1.71	1.430	1.115	0.983
2.4	1	1	1.571	1.740	2.174	2.679	3.229	3.812	4.083
	2	1	0.972	1.153	1.498	1.621	1.512	1.362	1.307
	3	1	1.377	1.470	1.653	1.612	1.363	1.079	0.963
3.2	1	1	2.503	2.618	3.053	3.855	5.093	6.453	7.063
	2	1	1.015	1.211	1.619	1.863	1.893	1.963	1.990
	3	1	1.029	1.197	1.604	1.585	1.424	1.218	1.146

TABLE 10

Normal ($K_b = \frac{E_1 b^2}{h^3 E_L}$) and Shear ($K_s = \frac{E_{12} b^2}{h^3 E_L}$) Buckling loads of square specially orthotropic 6668 plates (Mahogany plywood $D'_{11}/D'_{22} = 3.04$, $D'_{12}/D'_{22} = 0.438$, $E_L = 1.35 \times 10^6$ p.s.i)

Angle of orthotropy degrees	NORMAL BUCKLING COEFF. K_b		SHEAR BUCKLING COEFF. K_s	
	From Eqn. 2.20	From Reference 16	From equation 2.20	From Reference 16
0	1.080	1.08	2.566	2.56
			-2.566	-2.56
15	1.209	1.21	2.241	2.24
			-3.607	-3.61
30	1.494	1.50	2.399	2.41
			-4.900	-4.89
45	1.645	1.65	2.626	2.63
			-5.511	-5.50
60	1.494	1.50	2.399	2.41
			-4.900	-4.89
75	1.209	1.21	2.241	2.24
			-3.607	-3.61
90	1.080	1.08	2.566	2.56
			-2.566	-2.56

TABLE 11

Shear Buckling loads $K_s = \frac{N_{12}b^2}{b^2 E_L}$ of generally orthotropic
 SSSS plates (Mahogany plywood $D_{11}'/D_{22}' = 3.04$, $D_{126}'/D_{22}' =$
 0.438 , $E_L = 1.35 \times 10^6$ p.s.i.)

Angle of orthotro- picity degrees	SHEAR BUCKLING COEFFICIENTS K_s		
	$a/b \approx 0.5$	$a/b \approx 1.0$	$a/b \approx 1.5$
0	-9.731	-2.566	-1.579
	9.731	2.566	1.579
15	-13.243	-3.607	-2.127
	7.778	2.241	1.504
30	-15.392	-4.900	-3.130
	6.951	2.399	1.794
45	-13.805	-5.511	-3.968
	6.656	2.625	1.848
60	-10.200	-4.900	-4.046
	6.571	2.399	1.824
75	-7.000	-3.607	-3.394
	6.038	2.241	1.980
90	-5.735	-2.566	-2.464
	5.735	2.566	2.464

TABLE 12

Buckling loads ($K_b = N_1 b^2 / \pi^2 D_{22}^2$) of generally orthotropic plates with various boundary conditions (Maple plywood $D_{11}^2/D_{22}^2 = 3.117$, $D_{12}^2/D_{22}^2 = 0.12$, $D_{66}^2/D_{22}^2 = .26$)

Boun- dary condi- tions	Side ratio a/b	ANGLE OF ORTHOTROPICITY						
		0	15	30	45	60	75	90
C C C	0.5	52.571	48.403	38.429	28.247	22.181	20.542	20.621
	1.0	17.915	17.066	15.660	14.925	14.767	14.725	14.715
	1.5	14.967	14.085	13.173	13.340	13.049	12.047	11.492
	2.0	13.787	11.788	10.588	11.815	12.545	12.010	11.569
	2.5	11.819	10.048	9.717	12.054	13.794	13.992	13.837
	3.0	11.165	9.824	10.164	13.339	16.095	17.235	17.480
	3.5	11.519	10.546	11.380	15.288	19.163	21.453	22.195
	4.0	12.566	11.875	13.101	17.793	22.879	26.522	27.851
S S S	0.5	15.350	15.163	14.344	12.775	10.990	9.760	---
	1.0	9.866	10.351	11.750	12.135	10.839	10.195	---
	1.5	10.126	10.763	12.287	11.734	11.444	10.256	---
	2.0	9.822	9.794	9.591	11.009	11.868	11.398	---
	2.5	9.561	8.925	9.606	12.079	13.268	14.247	---
	3.0	9.391	9.451	10.638	13.574	16.768	18.242	---
	3.5	10.562	10.711	12.268	16.249	20.652	23.199	---
	4.0	11.519	11.875	13.101	17.793	22.879	26.522	27.851
S S S	0.5	14.026	13.570	12.185	9.997	7.602	6.247	6.076
	1.0	8.416	5.560	6.113	6.517	6.134	5.565	5.416
	1.5	4.933	5.105	6.160	7.629	6.432	5.123	4.829
	2.0	5.416	6.120	6.342	6.934	6.812	5.843	4.932
	2.5	4.884	5.522	6.621	7.102	7.453	6.121	4.903
	3.0	4.932	5.121	6.481	7.213	6.632	5.932	5.415
	3.5	9.948	5.231	6.302	7.314	6.881	5.821	---

Boundary conditions	Side ratio a/b	ANGLE OF ORTHOTROPICITY						
		0	15	30	45	60	75	90
F S S	0.5	12.807	11.695	9.069	6.447	4.878	4.365	---
	1.0	3.431	3.120	2.670	2.261	1.869	--	---
	1.5	1.698	1.487	1.227	1.339	1.308	0.940	---
	2.0	1.093	0.768	0.428	0.876	1.090	0.755	---
	2.5	0.814	.321	0.099	0.598	0.598	0.570	---
	3.0	0.662	---	0.165	0.288	0.007	0.624	---
C F F	3.5	0.572	0.206	---	0.075	.865	0.594	---
	0.5	3.122	2.722	1.967	1.307	0.983	0.957	1.000
	1.0	0.780	0.599	0.391	0.280	0.288	0.238	0.280
	1.5	0.347	0.188	0.078	0.086	0.101	0.105	0.110
	2.0	0.195	0.027	---	0.015	0.054	0.058	0.062
	2.5	0.126	---	---	---	0.032	0.037	0.040
F F F	3.0	0.087	---	---	---	0.020	0.025	0.028
	3.5	0.064	---	.157	---	0.013	0.018	0.020

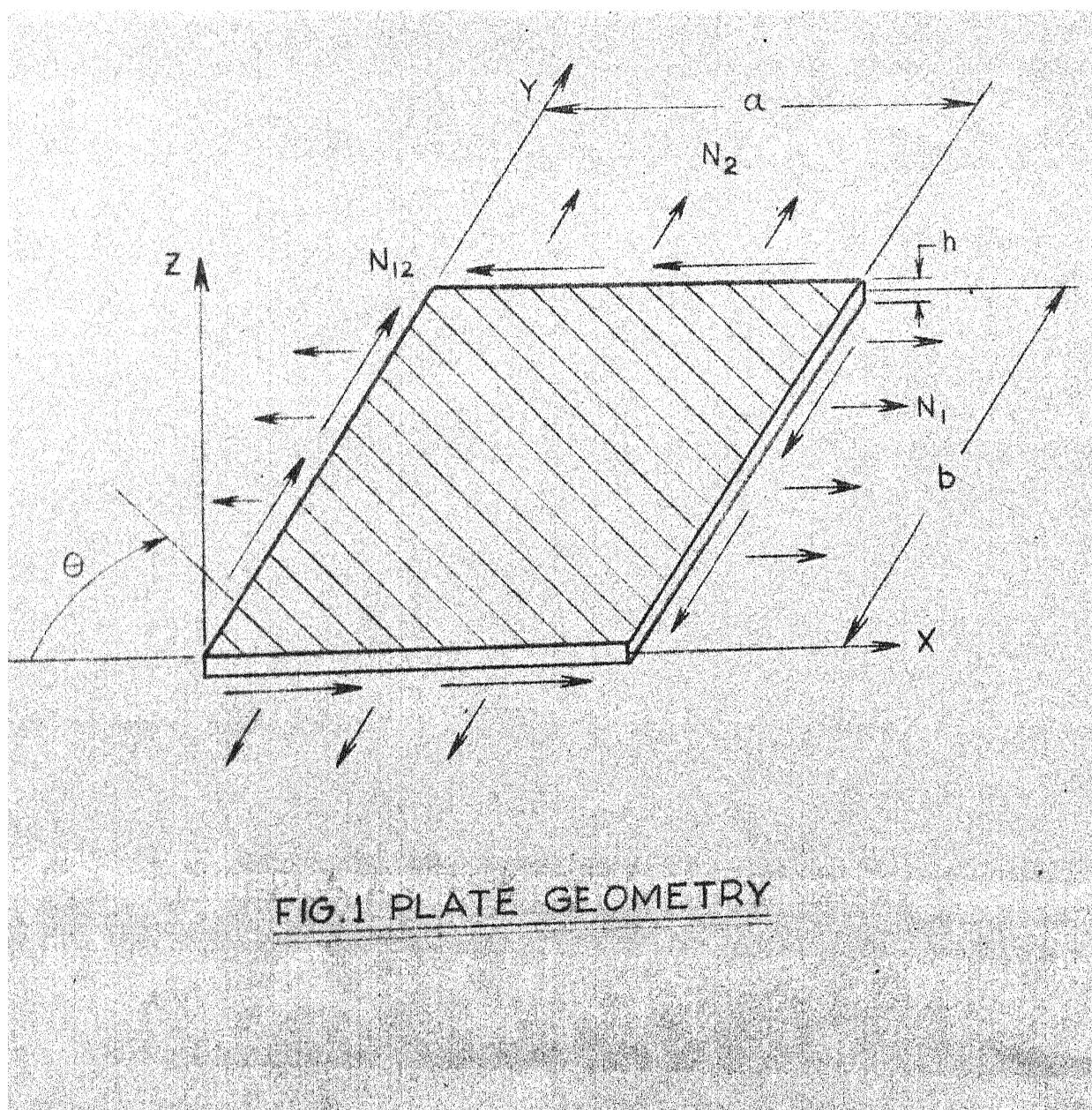
TABLE 13

Buckling loads ($K_b = N_1 b^2 / \pi^2 D_{22}^{'}$) of special orthotropic plates (Maple Plywood $D_{11}^{'}/D_{22}^{'} = 3.117$, $D_{12}^{'}/D_{22}^{'} = .12$, $D_{66}^{'}/D_{22}^{'} = .26$)

Angle of Orthotropicity Degrees	Side ratio a/b	CCCC		SSCC		SSSS	
		From Eqn. 3.34	From Ref. 16	From Eqn. 3.34	From Ref. 16	From Eqn. 3.34	From Ref. 16
0	0.6	52.571	52.648	15.350	15.541	14.036	14.026
	1.0	17.915	18.208	9.86	10.181	5.416	5.416
	1.5	14.967	15.499	10.126	10.275	4.933	4.933
	2.0	13.787	12.391	10.822	10.181	5.416	5.416
	2.5	11.815	11.260	9.861	9.925	4.854	4.855
	3.0	11.165	11.243	9.891	10.181	4.932	4.932
	3.5	11.619	11.725	10.862	11.280	4.948	4.948

APPENDIX - D

FIGURES



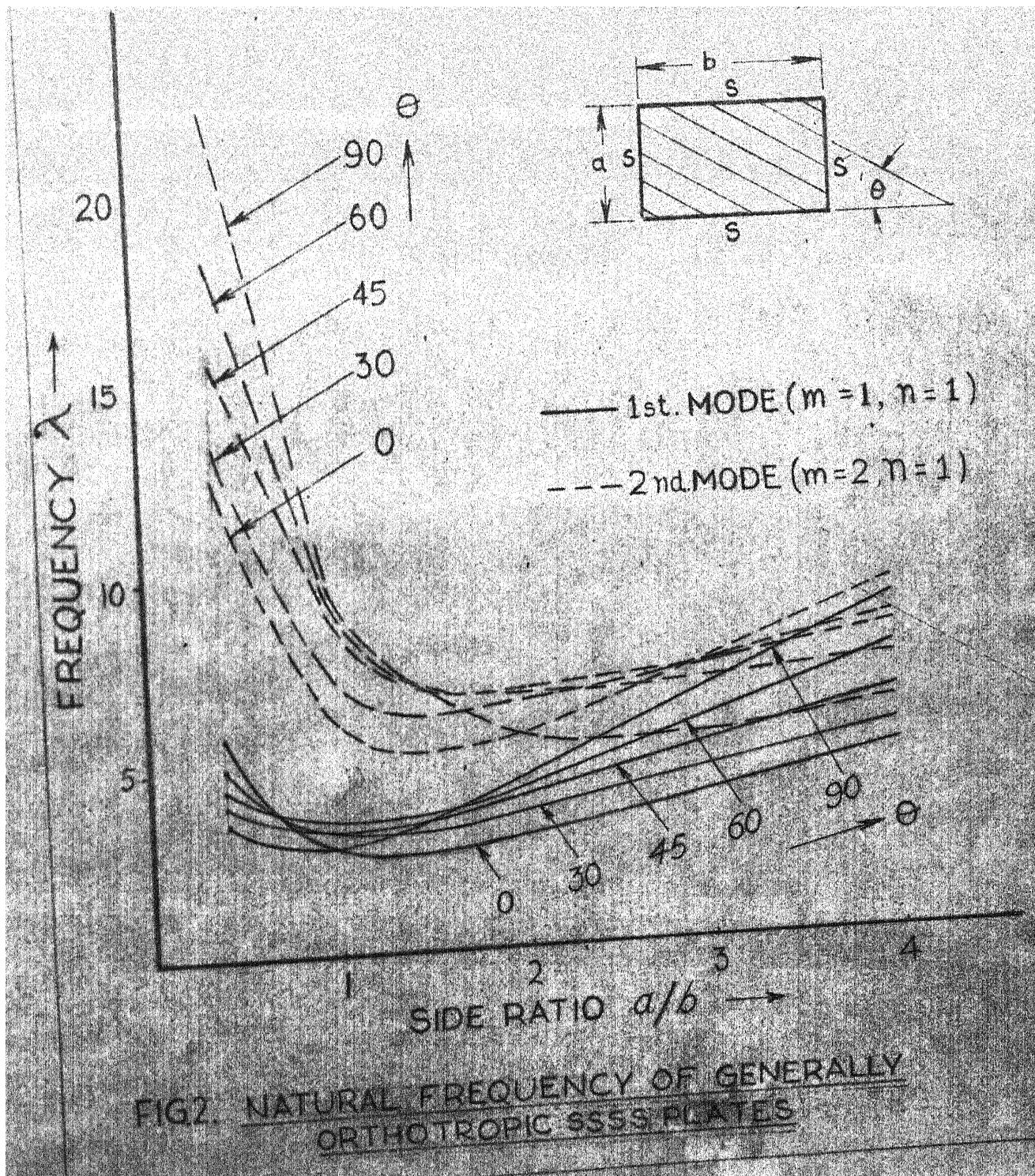


FIG2. NATURAL FREQUENCY OF GENERALLY
ORTHOTROPIC SSSS PLATES

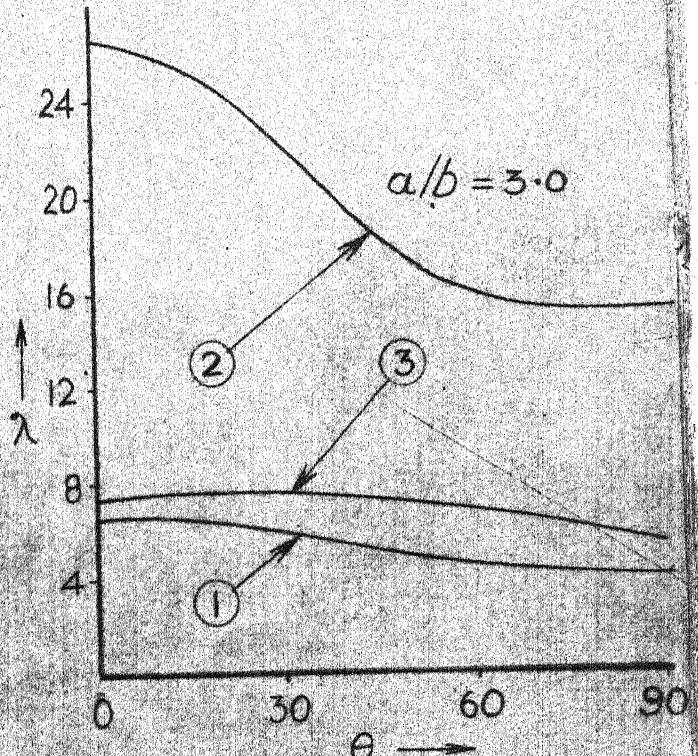
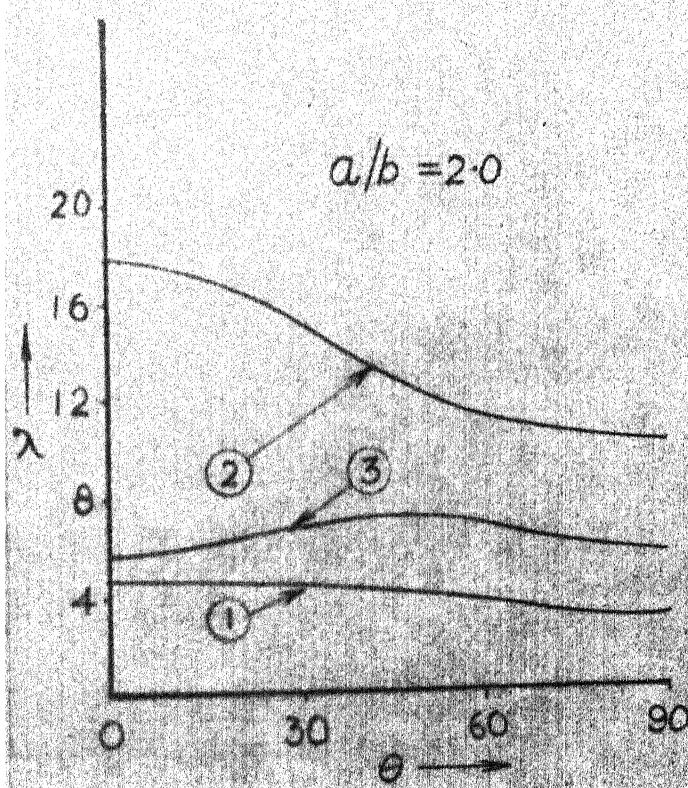
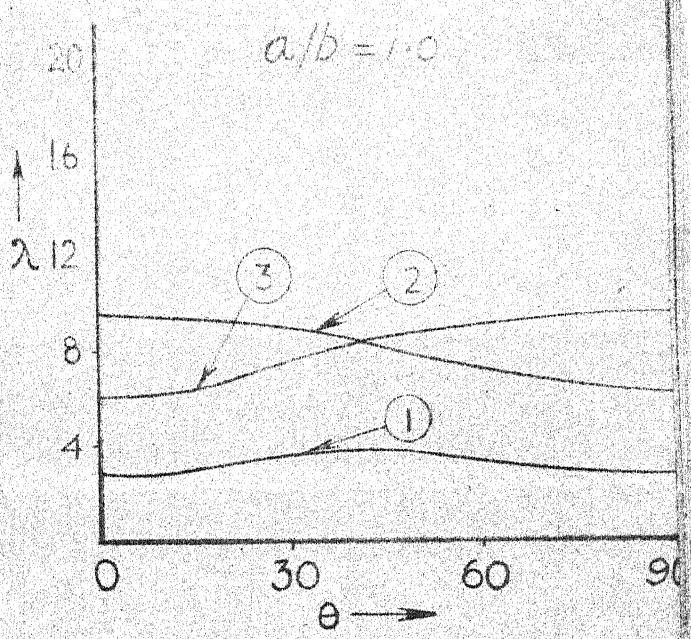
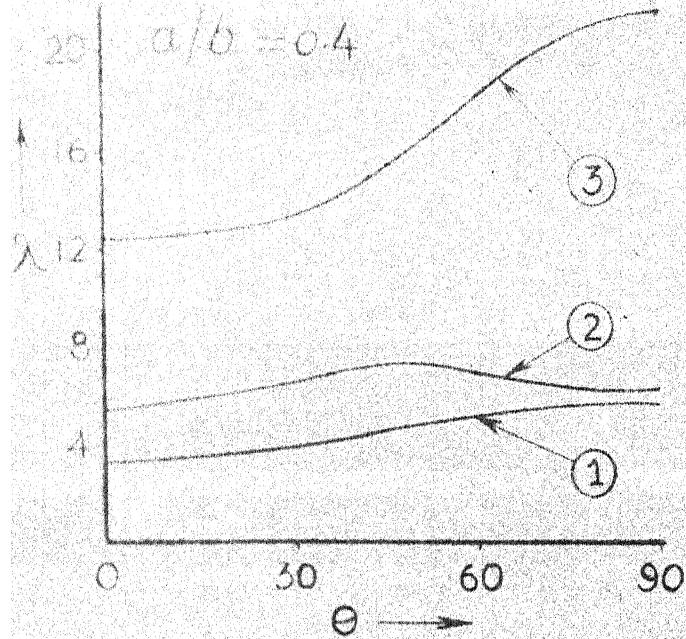
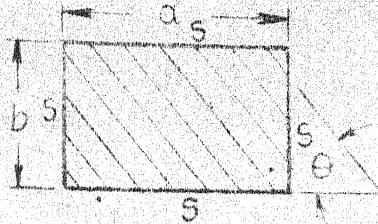
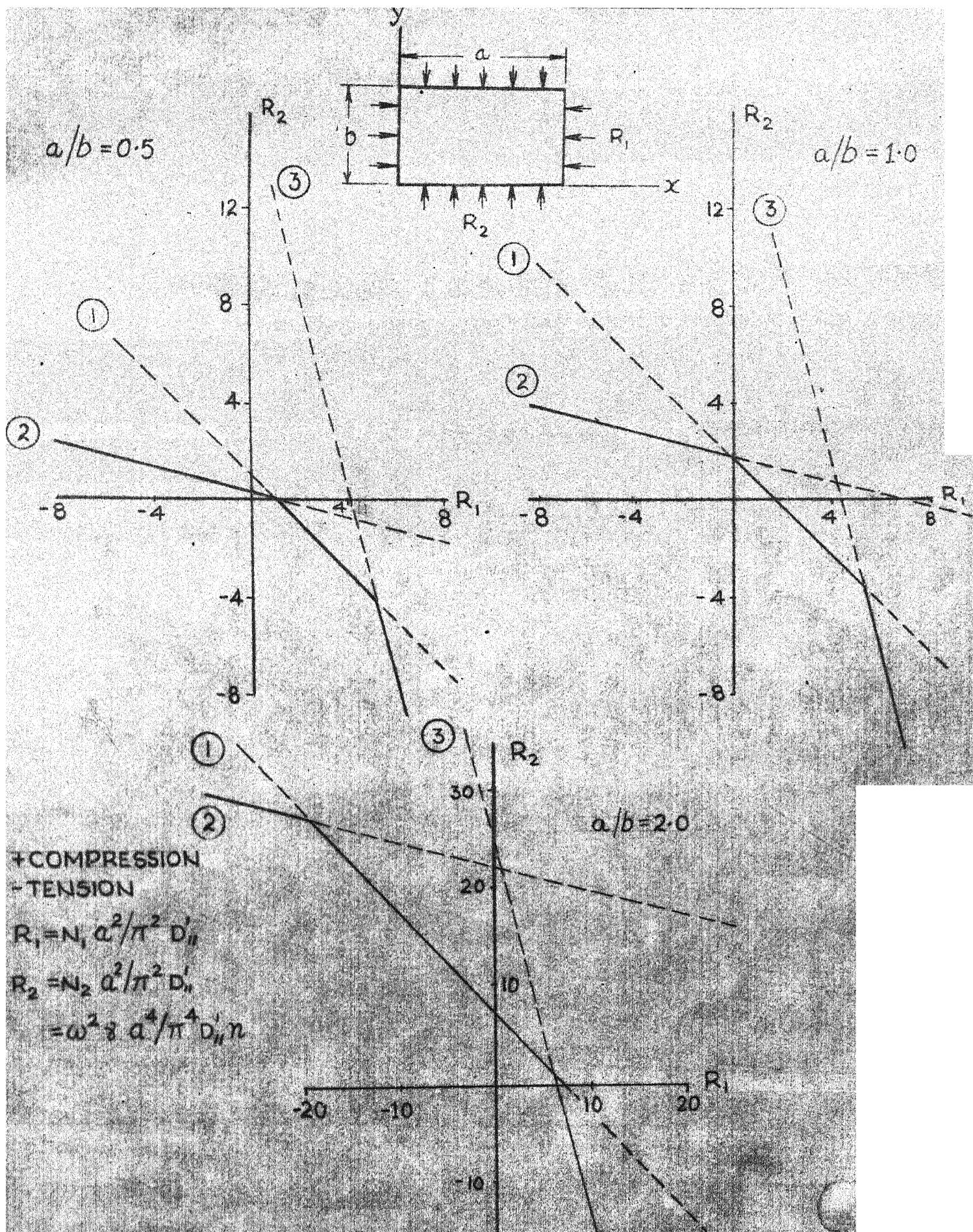


FIG.3 NATURAL FREQUENCY OF GENERALLY
ORTHOTROPIC SSSS PLATES

① $\rightarrow m=1, n=1$; ② $\rightarrow m=1, n=2$; ③ $\rightarrow m=2, n=1$.



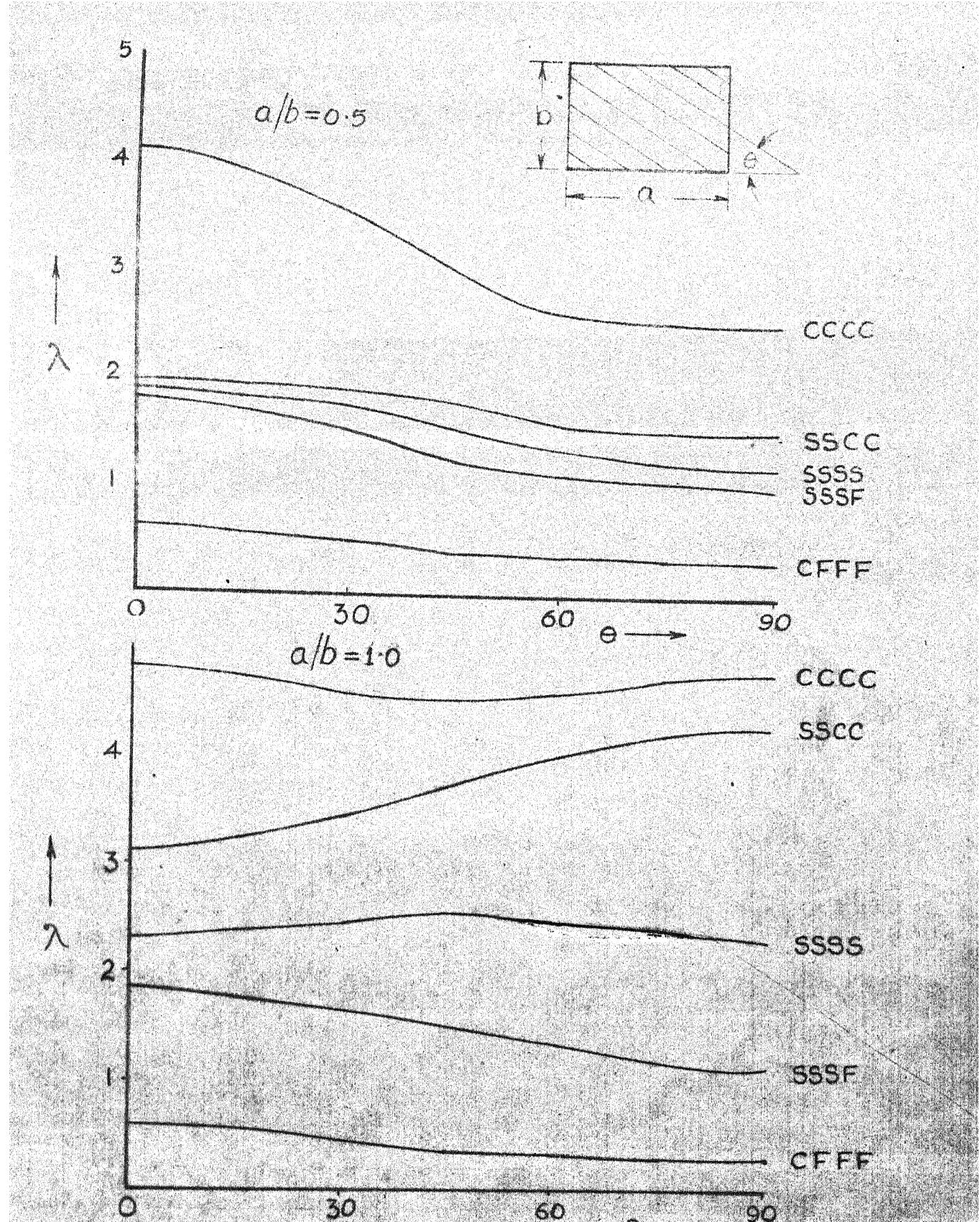


FIG. 5 NATURAL FREQUENCY OF GENERALLY ORTHOTROPIC PLATES WITH VARIOUS B.C.

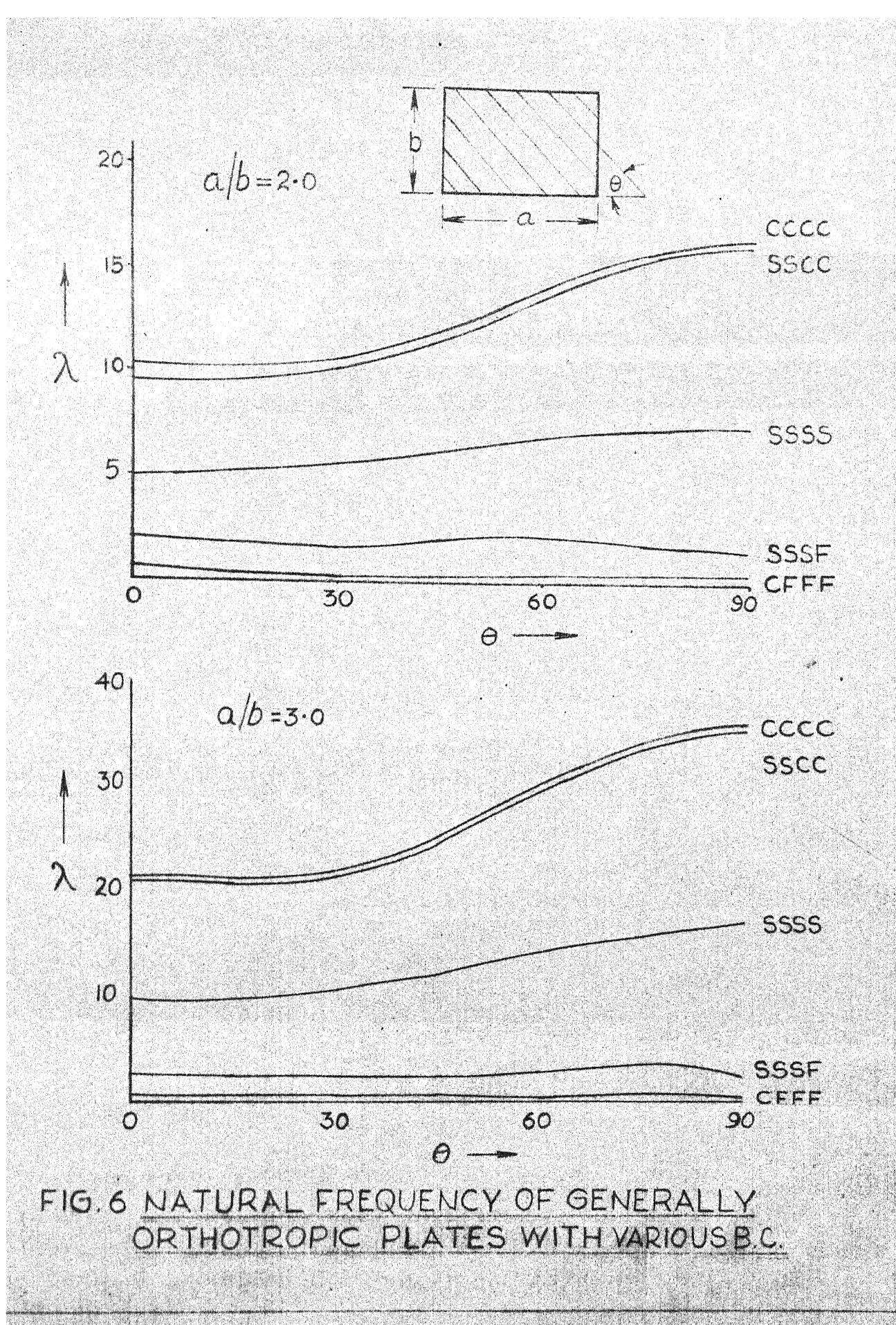


FIG. 6 NATURAL FREQUENCY OF GENERALLY ORTHOTROPIC PLATES WITH VARIOUS B.C.

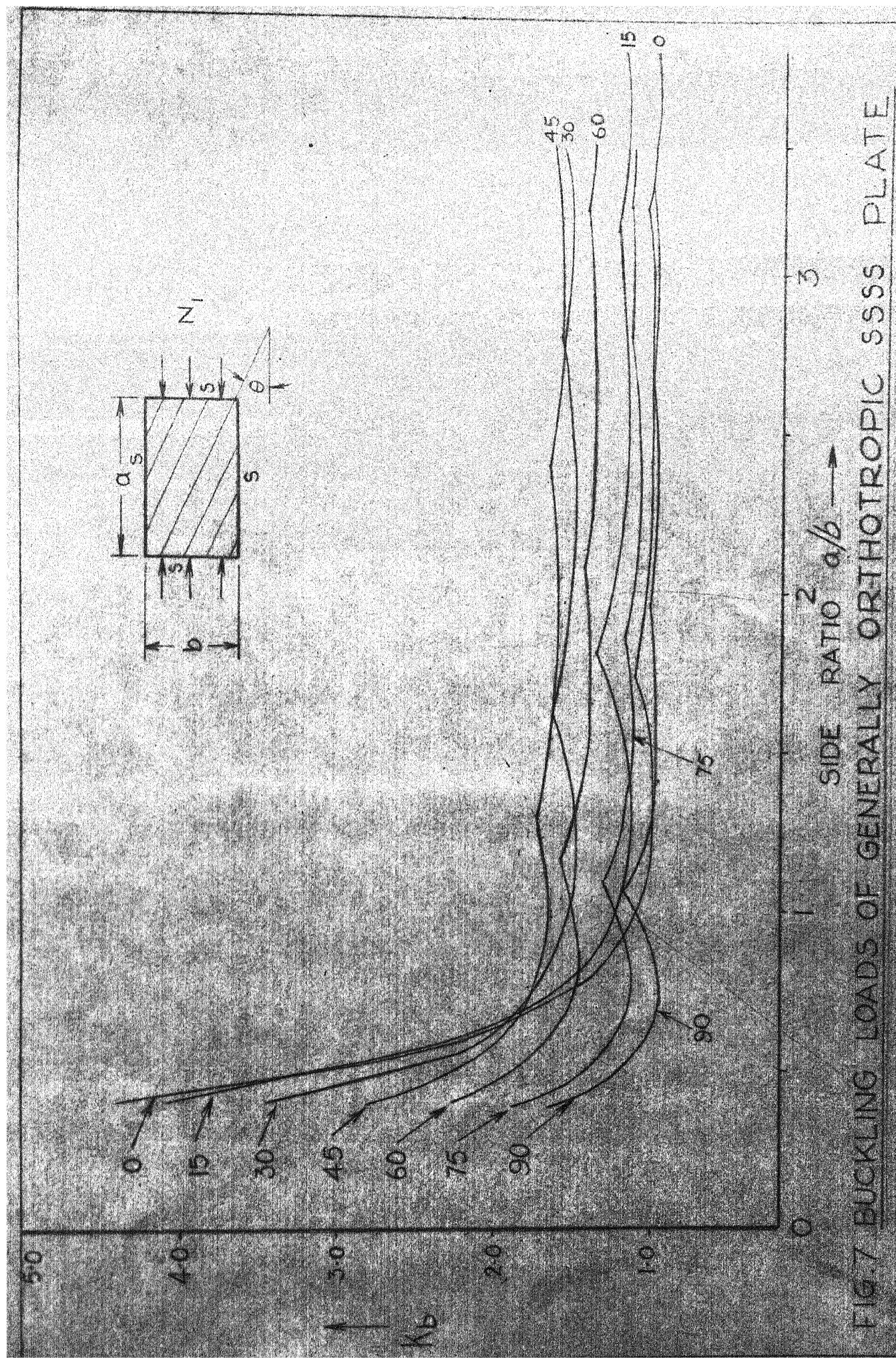


FIG. 7 BUCKLING LOADS OF GENERALLY ORTHOTROPIC SSSS PLATE

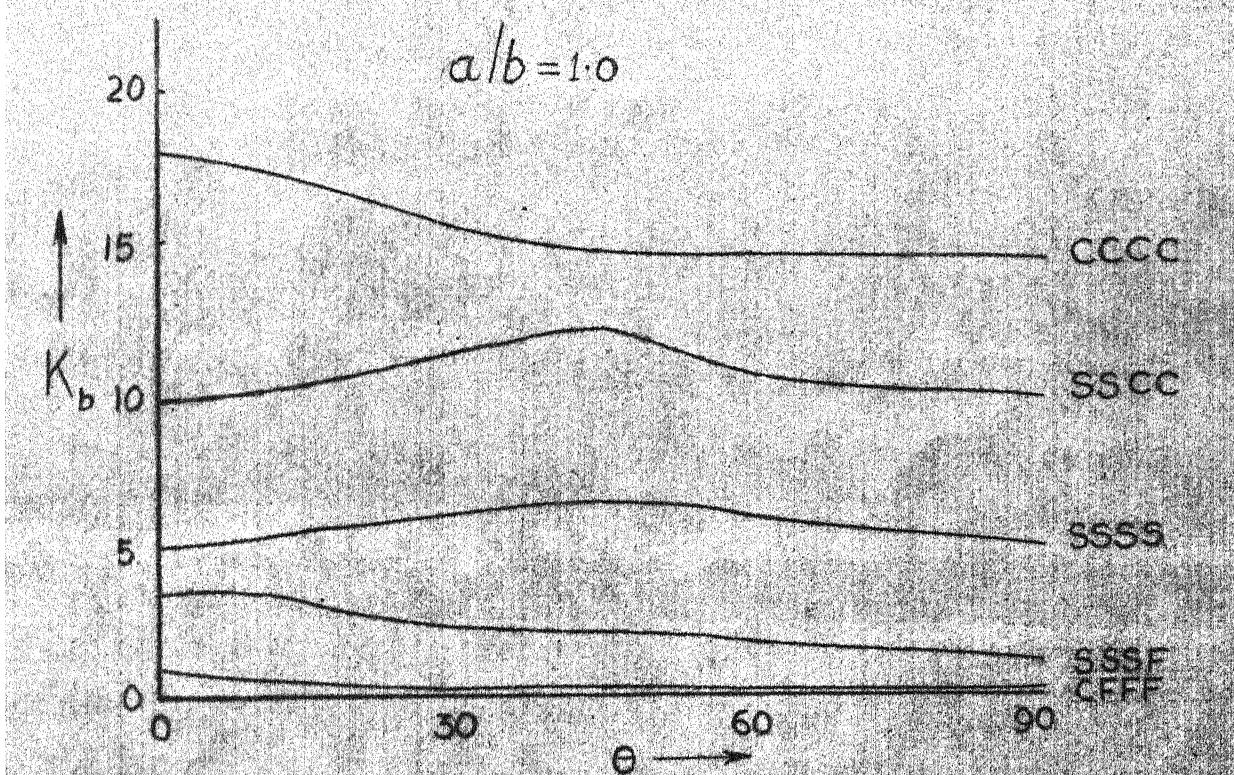
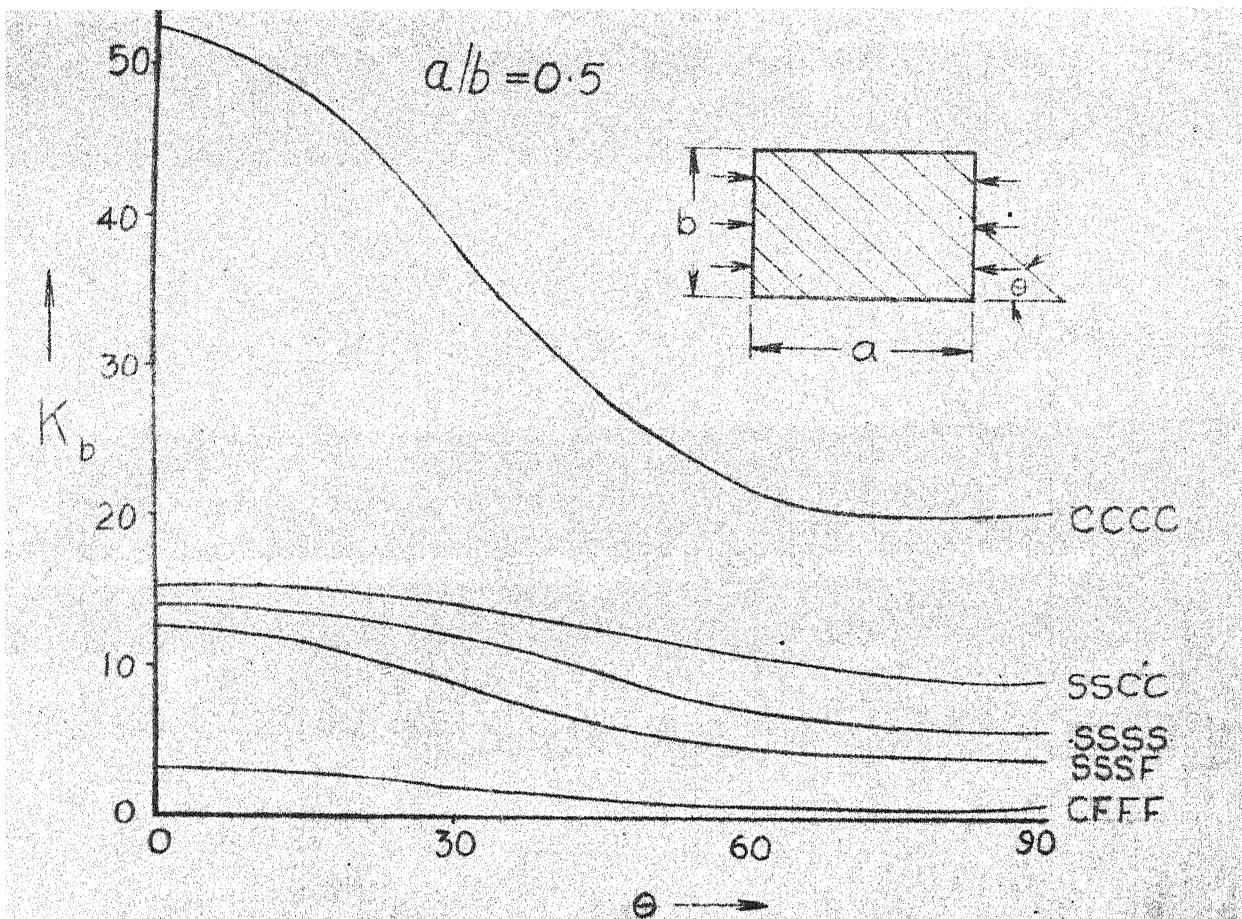


FIG. 8 BUCKLING LOADS OF GENERALLY ORTHOTROPIC PLATES WITH VARIOUS B.C.

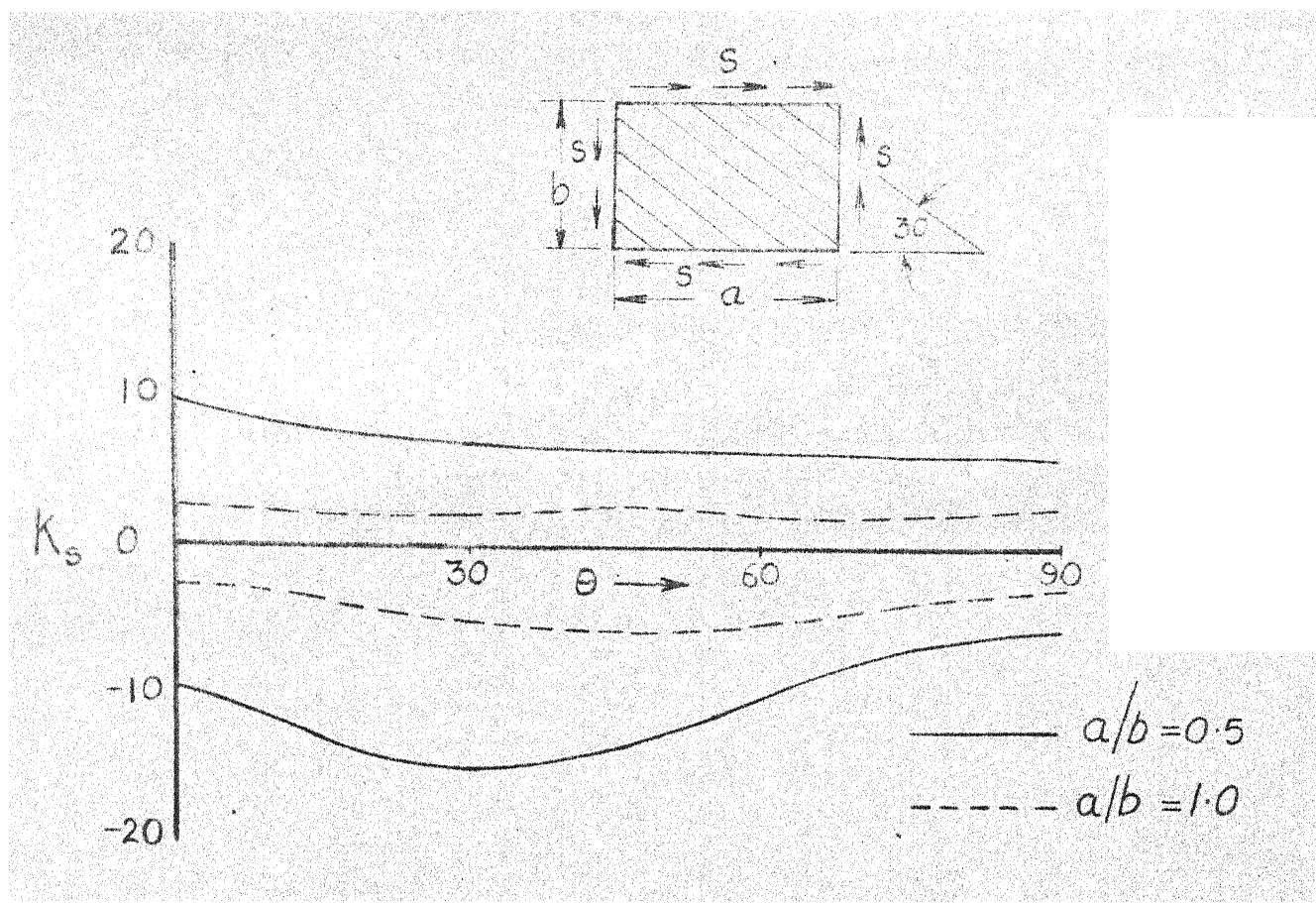


FIG. 9 SHEAR BUCKLING LOADS OF GENERALLY
ORTHOTROPIC SSSS PLATES

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Swarup,
Vibration and buckling of generally orthotropic plates.

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